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**Absorptive nature of scattering coefficients in stress-energy tensor formalism for room acoustics**

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1 In the stress-energy tensor formalism, the symmetry between absorption and scatter-  
2 ing coefficients, as proven by measurements combined with simulations, is counter-  
3 intuitive. By introducing the wall admittance, we show that the scattering coefficient  
4 is partly created by the real part of the wall admittance combined with the active  
5 intensity, that is, is partly due to absorption. However, for curved surfaces or finite  
6 source distances, it also depends on the imaginary part of the wall admittance in  
7 combination with the reactive intensity, which confers it genuine scattering proper-  
8 ties inversely proportional to the distances to the sources. Thus, for plane waves  
9 impinging on plane boundaries, or purely real admittances, scattering reduces to  
10 absorption.

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## 11 I. INTRODUCTION

12 The stress-energy tensor formalism was initially introduced by Morse and collaborators  
13 (Morse and Feshbach, 1956; Morse and Ingard, 1968) in order to describe energy conservation  
14 in linear acoustics. Many years later, Stanzial and collaborators applied the conservation  
15 of the stress-energy tensor to room acoustics (Stanzial *et al.*, 2002), focusing on the expres-  
16 sion of the radiation pressure. In a subsequent paper (Stanzial and Schiffrer, 2010), they  
17 introduced the energy velocity as ratio of the sound intensity to the total acoustic energy,  
18 and derived from the conservation of the stress-energy tensor an expression for the local  
19 initial reverberation time at the onset of energy decay. Some related ideas, linking wave  
20 impedance to the complex sound intensity, are also found in (Mann *et al.*, 1987; Stanzial  
21 and Graffigna, 2017). Mann *et al.* focused on energy transfer and analysed in terms of  
22 energy flux and power what happens in the vicinity of intensity vortex that are created  
23 by specific configurations of sources. Stanzial and Graffigna went one step further in the  
24 definition and measurement of complex intensity. However, both papers are outside the  
25 scope of the present analysis that focuses on the active stress-energy tensor - see (Polack,  
26 2023) for an extension to the *complex* stress-energy tensor and its conservation, together  
27 with preliminary measurement results.

28 A somewhat different perspective was introduced by Dujourdy *et al.* (Dujourdy *et al.*,  
29 2017; 2019) in order to generalize the diffusion equation formalism of Ollendorf and Picaut  
30 (Ollendorff, 1969; Picaut *et al.*, 1997). Indeed, where the diffusion equation arbitrarily  
31 introduces a gradient type relationship between sound intensity and total energy, the stress-

32 energy tensor formalism introduces instead the conservation equation for intensity. When  
 33 integrated in disproportionate enclosures, the stress-energy tensor yields both absorption  
 34 and scattering on the boundaries. Dujourdy *et al.* stressed the symmetrical role played  
 35 by absorption and scattering coefficients in the conservation equations, which was then  
 36 systematically investigated and confirmed by Meacham *et al.* (Meacham *et al.*, 2019). The  
 37 present papers aims at proving this symmetry and the absorbing nature of the scattering  
 38 coefficient, using the local admittance formalism at the boundaries.

39 After presenting the background of the stress-energy conservation, we successively express  
 40 the elements of the stress energy tensor on the boundaries, then the absorption and scattering  
 41 coefficients, in terms of the local admittance. We then examine the main types of boundaries,  
 42 and discuss the relationship between absorption and scattering coefficients.

## 43 II. THE BASIC EQUATIONS

44 Dujourdy *et al.* (Dujourdy *et al.*, 2017, 2019) have shown that the wave equation, satisfied  
 45 by the velocity potential, can be extended by a set of conservation equations that reduces  
 46 to the conservation of the stress-energy tensor  $\underline{T}$ .

47 Let  $\psi$  be the velocity potential. The conservation of the stress-energy tensor is expressed  
 48 as:

$$\vec{\nabla} \cdot \underline{T} = 0 \tag{1}$$

49 with

$$\underline{\underline{T}} = \begin{pmatrix} E_{tt} & E_{tx} & E_{ty} & E_{tz} \\ E_{tx} & E_{xx} & E_{xy} & E_{xz} \\ E_{ty} & E_{xy} & E_{yy} & E_{yz} \\ E_{tz} & E_{xz} & E_{yz} & E_{zz} \end{pmatrix}$$

50 In terms of the velocity potential, the elements of the stress-energy tensor are given by:

$$E_{tt} = \frac{\rho}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\vec{\nabla} \psi|^2 \right)$$

$$E_{tx} = -\frac{\rho_0}{c} \partial_t \psi \partial_x \psi$$

$$E_{ty} = -\frac{\rho_0}{c} \partial_t \psi \partial_y \psi$$

$$E_{tz} = -\frac{\rho_0}{c} \partial_t \psi \partial_z \psi$$

$$E_{xx} = \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\partial_x \psi|^2 - |\partial_y \psi|^2 - |\partial_z \psi|^2 \right)$$

$$E_{yy} = \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 - |\partial_x \psi|^2 + |\partial_y \psi|^2 - |\partial_z \psi|^2 \right)$$

$$E_{zz} = \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 - |\partial_x \psi|^2 - |\partial_y \psi|^2 + |\partial_z \psi|^2 \right)$$

$$E_{xy} = \rho_0 \partial_x \psi \partial_y \psi$$

$$E_{xz} = \rho_0 \partial_x \psi \partial_z \psi$$

$$E_{yz} = \rho_0 \partial_y \psi \partial_z \psi$$

51 where  $\rho_0$  is the steady-state density of air and  $c$  the speed of sound.

52 In the specific cases of long enclosures (Dujourdy *et al.*, 2017) and flat spaces (Dujourdy  
53 *et al.*, 2019), Dujourdy *et al.* were able to reduce the conservation of the stress-energy tensor  
54 (eq. 1) to the telegraphers equation:

$$\frac{1}{c^2} \partial_{tt} E - \partial_{xx} E + \frac{A+D}{\lambda c} \partial_t E + \frac{AD}{\lambda^2} E = 0 \quad (2)$$

55 where  $E$  is the total energy,  $\lambda$  is the mean free path of the enclosure, and  $A$  and  $D$  are  
 56 respectively modified absorption and scattering coefficients at the boundaries. Eq. (2) is  
 57 symmetrical with respect to the absorption and scattering coefficients, as was confirmed by  
 58 Meacham *et al.* (Meacham *et al.*, 2019) who systematically explored the whole space of values  
 59 for the absorption and scattering coefficients and compared thus obtained reverberation  
 60 times and spatial decays with the measured ones in a hallway. As shown in Fig. 1, the  
 61 measured reverberation times and spatial decays correspond to narrow stripes of values for  
 62  $A$  and  $D$ , which do not always overlap; but most striking in Fig. 1 is the symmetry of the  
 63 figure with respect to absorption and scattering coefficients. Physical justification of this  
 64 symmetry is therefore the goal of the present paper.

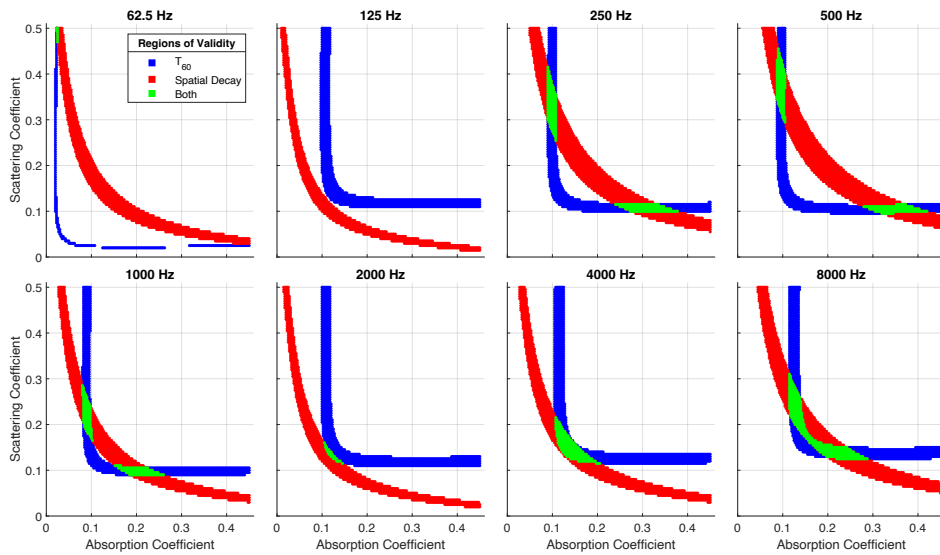


FIG. 1. Relevant combinations of absorption and scattering coefficients leading to measured reverberation times and spatial decays in a hallway (reproduced from (Meacham *et al.*, 2019)).

65 **III. WALL ADMITTANCE**

66 Now, wall admittance introduces a relation between the normal derivative and the time  
 67 derivative of the velocity potential on the wall:

$$-\partial_n \psi = \frac{\beta}{c} \partial_t \psi = \frac{1}{c\zeta} \partial_t \psi$$

68 with  $\vec{n}$  the exterior normal to the wall,  $\zeta$  the specific acoustic impedance of the wall ([Morse](#)  
 69 [and Ingard, 1968](#), p. 580), that is, the wall impedance normalized by the characteristic  
 70 impedance  $\rho_0 c$  of air, and  $\beta = \frac{1}{\zeta}$  is the specific acoustic admittance. Let us then consider  
 71 a flat enclosure with vertical coordinate  $z$ . We can express the elements  $E_{\cdot z}$  of the stress-  
 72 energy tensor, where  $\cdot$  takes the values  $t, x,$  or  $y$ . However, the admittance formalism implies  
 73 a complex velocity potential, whereas the elements of the stress-energy tensor must remain  
 74 real, as is expected from energy quantities. Thus, the expression for the non-diagonal terms  
 75 of the stress-energy tensor must be modified as:

$$E_{tx} = -\frac{\rho_0}{c} \Re(\partial_t \psi \partial_x \psi^*)$$

$$E_{ty} = -\frac{\rho_0}{c} \Re(\partial_t \psi \partial_y \psi^*)$$

$$E_{tz} = -\frac{\rho_0}{c} \Re(\partial_t \psi \partial_z \psi^*)$$

$$E_{xy} = \rho_0 \Re(\partial_x \psi \partial_y \psi^*)$$

$$E_{xz} = \rho_0 \Re(\partial_x \psi \partial_z \psi^*)$$

$$E_{yz} = \rho_0 \Re(\partial_y \psi \partial_z \psi^*)$$

76 where the exponent  $*$  means complex conjugated.

77 We thus obtain on the "ceiling":

$$\begin{aligned}
 E_{tz} &= \frac{\rho_0}{c^2} \Re(\partial_t \psi \beta^* \partial_t \psi^*) = \frac{\rho_0}{c^2} \Re(\beta) |\partial_t \psi|^2 \\
 E_{xz} &= -\frac{\rho_0}{c} \Re(\partial_x \psi \beta^* \partial_t \psi^*) = \Re(\beta) E_{tx} + \Im(\beta) \Im\left(\frac{\rho_0}{c} \partial_x \psi \partial_t \psi^*\right) \\
 E_{yz} &= -\frac{\rho_0}{c} \Re(\partial_y \psi \beta^* \partial_t \psi^*) = \Re(\beta) E_{ty} + \Im(\beta) \Im\left(\frac{\rho_0}{c} \partial_y \psi \partial_t \psi^*\right)
 \end{aligned} \tag{3}$$

78 and the same equations with opposite sign on the "floor". Note that  $E_{xz}$  and  $E_{yz}$  correspond  
 79 to the parallel components of the radiation pressure exerted on the "ceiling" and the "floor"  
 80 ([Stanzial \*et al.\*, 2002](#)).

#### 81 IV. INTERPRETATION

82 Recalling that  $E_{tx}$  and  $E_{ty}$  are the components of the sound intensity in the  $x$  and  
 83  $y$  directions, we observe that the boundary conditions for  $E_{xz}$  and  $E_{yz}$  involve not only  
 84 the active intensity associated with the real part of the admittance, but also the reactive  
 85 intensity associated with the imaginary part of the admittance. This is not surprising, as  
 86 the reactive intensity corresponds to local recirculation of energy, just as the imaginary part  
 87 of the wall admittance is associated with a non-flat surface. Both phenomena contribute to  
 88 the scattering of the impinging waves.

89 However, recalling that admittances are defined for pure waves only, we can further sim-  
 90 plify these expressions by considering the phase angles  $\phi_x$  and  $\phi_y$  between the space deriva-  
 91 tives of the velocity potential with respect to  $x$  and  $y$  respectively, and its time derivative.



92 The last two ceiling equations then reduce to:

$$E_{xz} = [\Re(\beta) + \Im(\beta) \tan \phi_x] E_{tx}$$

$$E_{yz} = [\Re(\beta) + \Im(\beta) \tan \phi_y] E_{ty}$$

93 As for  $E_{tz}$ , we must first express  $E_{tt}$  as a function of the wall admittance, that is:

$$\begin{aligned} E_{tt} &= \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\vec{\nabla} \psi|^2 \right) \\ &= \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\partial_x \psi|^2 + |\partial_y \psi|^2 + |\partial_z \psi|^2 \right) \\ &= \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\partial_x \psi|^2 + |\partial_y \psi|^2 + \left| \frac{\beta}{c} \partial_t \psi \right|^2 \right) \\ &= \frac{\rho_0}{2} \left( \frac{1}{c^2} [1 + |\beta|^2] |\partial_t \psi|^2 + |\partial_x \psi|^2 + |\partial_y \psi|^2 \right) \end{aligned}$$

94 Introducing the angle of incidence  $\theta$  of the wave makes it possible to express the last two

95 terms in the previous equation as:

$$|\partial_x \psi|^2 + |\partial_y \psi|^2 = \frac{\sin^2 \theta}{c^2} |\partial_t \psi|^2$$

96 that is:

$$E_{tt} = \frac{\rho_0}{2c^2} (1 + \sin^2 \theta + |\beta|^2) |\partial_t \psi|^2 \quad (4)$$

97 leading to the following expression for the absorption coefficient:

$$\alpha(\theta) = \frac{E_{tz}}{E_{tt}} = \frac{2 \Re(\beta)}{1 + \sin^2 \theta + |\beta|^2} \quad (5)$$

98 The interpretations of the scattering coefficients  $D_x$  and  $D_y$  are similar. We simply

99 obtain:

$$D_x = \frac{E_{xz}}{E_{tx}} = [\Re(\beta) + \Im(\beta) \tan \phi_x] \quad (6)$$

$$D_y = \frac{E_{yz}}{E_{ty}} = [\Re(\beta) + \Im(\beta) \tan \phi_y]$$

100 **V. BOUNDARY CONDITIONS**

101 The evaluation of eqs. (6) takes different forms depending on the type of boundary and  
 102 the position of the source.

103 **A. Plane wave on flat boundary**

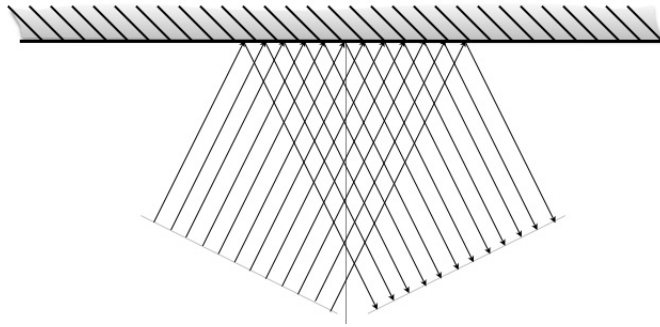


FIG. 2. Plane wave reflecting on flat boundary.

104 For a plane wave impinging on a flat boundary  $z = 0$  (Fig. 2), the velocity potential  
 105 takes the following form:

$$\psi(t, r) = \exp -i(\omega t - k_x x - k_y y - k_z z) + R \exp -i(\omega t - k_x x - k_y y + k_z z) \quad (7)$$

106 where  $\omega$  is the radian frequency,  $k_x$ ,  $k_y$  and  $k_z$  the components of the wave number  $k = \frac{\omega}{c}$   
 107 along the three coordinates, and  $R$  the complex reflection coefficient on the boundary. The  
 108 first term in eq. (7) represents the impinging wave; and the second the reflected wave. The  
 109 time derivative in the neighbourhood of the boundary is given by:

$$\partial_t \psi = -i\omega \psi = -i\omega \exp -i(\omega t - k_x x - k_y y) [\exp i k_z z + R \exp -i k_z z]$$

110 and the space derivatives by:

$$\begin{aligned}\partial_x \psi &= ik_x \psi = ik_x \exp -i(\omega t - k_x x - k_y y) [\exp ik_z z + R \exp -ik_z z] = -\frac{k_x}{\omega} \partial_t \psi \\ \partial_y \psi &= ik_y \psi = ik_y \exp -i(\omega t - k_x x - k_y y) [\exp ik_z z + R \exp -ik_z z] = -\frac{k_y}{\omega} \partial_t \psi\end{aligned}\quad (8)$$

$$\partial_z \psi = ik_z \exp -i(\omega t - k_x x - k_y y) [\exp ik_z z - R \exp -ik_z z] = -\frac{\beta}{c} \partial_t \psi$$

111 As a consequence, on the boundary  $z = 0$ , the phase angles  $\phi_x$  and  $\phi_y$  between the space  
112 derivatives of the velocity potential with respect to  $x$  and  $y$  and its time derivative are both  
113 equal to 0. In this case, eqs. (6) simply reduce to:

$$D_x = D_y = \Re(\beta)$$

114 that is, the scattering coefficient is purely created by absorption. Note that the last line  
115 of eq. (8), together with the relation between the normal component of the wave number  
116 and the angle of incidence  $k_z = k \cos \theta = \frac{\omega}{c} \cos \theta$ , gives the expected relation between the  
117 complex reflection coefficient and the admittance:

$$R = \frac{\cos \theta - \beta}{\cos \theta + \beta} = \frac{[\cos^2 \theta - |\beta|^2] - 2i \cos \theta \Im \beta}{\cos^2 \theta + 2 \cos \theta \Re \beta + |\beta|^2}$$

## 118 B. Source at finite distance from flat boundary

119 For a spherical wave impinging on a flat boundary (Fig. 3), the velocity potential takes  
120 a different form:

$$\psi(t, r) = \frac{\exp -i(\omega t - kr)}{4\pi r} + R \frac{\exp -i(\omega t - kr')}{4\pi r'} \quad (9)$$

121 where  $R$  still is the complex reflection coefficient on the boundary,  $r$  the distance to the real  
122 source  $S$ , and  $r'$  the distance to the virtual image source  $S'$  (see Fig. 3). The first term in eq.  
123 (9) represents the impinging wave; and the second the reflected wave. The time derivative

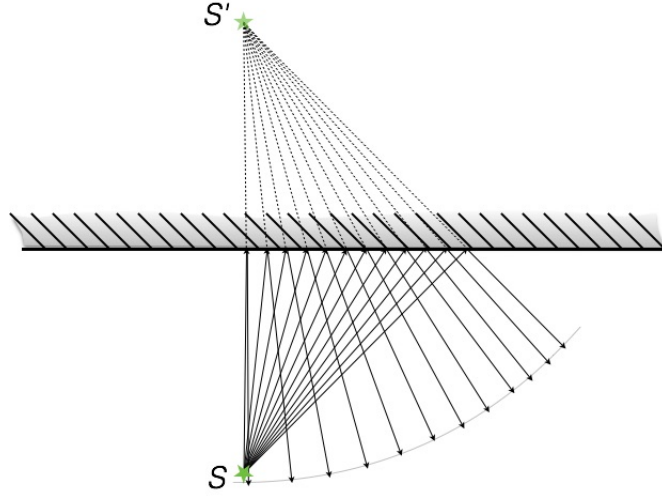


FIG. 3. Source at finite distance from flat boundary; S is real source, and S' virtual image source.

124 in the neighbourhood of the boundary is now given by:

$$\partial_t \psi = -i\omega \psi = -i\omega \frac{\exp -i\omega t}{4\pi} \left[ \frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'} \right] \quad (10)$$

125 and the space derivatives by:

$$\begin{aligned} \partial_x \psi &= i \frac{\exp -i\omega t}{4\pi} \left[ k_x \frac{\exp ikr}{r} \left( 1 + \frac{i}{kr} \right) + R k'_x \frac{\exp ikr'}{r'} \left( 1 + \frac{i}{kr'} \right) \right] \\ \partial_y \psi &= i \frac{\exp -i\omega t}{4\pi} \left[ k_y \frac{\exp ikr}{r} \left( 1 + \frac{i}{kr} \right) + R k'_y \frac{\exp ikr'}{r'} \left( 1 + \frac{i}{kr'} \right) \right] \\ \partial_z \psi &= i \frac{\exp -i\omega t}{4\pi} \left[ k_z \frac{\exp ikr}{r} \left( 1 + \frac{i}{kr} \right) + R k'_z \frac{\exp ikr'}{r'} \left( 1 + \frac{i}{kr'} \right) \right] \end{aligned} \quad (11)$$

126 where  $k_x$ ,  $k_y$  and  $k_z$  are the components of the incident wave number  $k$  along the three

127 coordinates, and  $k'_x$ ,  $k'_y$ , and  $k'_z$  those of the reflected wave number  $k'$ . At the boundary, the

128 distance to the real and imaginary sources are equal, therefore  $r = r'$ ; and  $k_x = k'_x$ ,  $k_y = k'_y$ ,

129 but  $k'_z = -k_z$ . The partial derivatives further reduce to:

$$\begin{aligned}
 \partial_t \psi &= -i\omega \frac{\exp -i(\omega t - kr)}{4\pi r} (1 + R) \\
 \partial_x \psi &= ik_x \frac{\exp -i(\omega t - kr)}{4\pi r} (1 + R) \left(1 + \frac{i}{kr}\right) = -\frac{k_x}{\omega} \left(1 + \frac{i}{kr}\right) \partial_t \psi \\
 \partial_y \psi &= ik_y \frac{\exp -i(\omega t - kr)}{4\pi r} (1 + R) \left(1 + \frac{i}{kr}\right) = -\frac{k_y}{\omega} \left(1 + \frac{i}{kr}\right) \partial_t \psi \\
 \partial_z \psi &= ik_z \frac{\exp -i(\omega t - kr)}{4\pi r} (1 - R) \left(1 + \frac{i}{kr}\right) = -\frac{\beta}{c} \partial_t \psi
 \end{aligned} \tag{12}$$

130 As a consequence, on the boundary, the phase angles  $\phi_x$  and  $\phi_y$  between the space derivatives  
 131 of the velocity potential with respect to  $x$  and  $y$  and its time derivative are both equal to  
 132  $\arctan \frac{1}{kr}$ . And eqs. (6) reduce to:

$$D_x = D_y = \Re(\beta) + \frac{\Im(\beta)}{kr}$$

133 In this case, the imaginary part of the admittance contributes to the scattering coefficient,  
 134 and this contribution is inversely proportional to  $kr$ . And according to the last line of eq.  
 135 (12), the usual relation between the complex reflection coefficient and the admittance is  
 136 modified into:

$$R = \frac{\cos \theta \left(1 + \frac{i}{kr}\right) - \beta}{\cos \theta \left(1 + \frac{i}{kr}\right) + \beta} = \frac{[\cos^2 \theta \left(1 + \frac{1}{k^2 r^2}\right) - |\beta|^2] - 2i \cos \theta \left[\Im \beta + \frac{\Re \beta}{kr}\right]}{\cos^2 \theta \left(1 + \frac{1}{k^2 r^2}\right) + 2 \cos \theta \left(\Re \beta + \frac{\Im \beta}{kr}\right) + |\beta|^2}$$

137 Note that this expression is equivalent to introducing a complex angle of incidence  $\theta$ .

### 138 C. Curved boundary

139 In the case of a curved boundary, the real and virtual sources are not located at the  
 140 same distance from the boundary (Fig. 4). As a consequence, eqs. (10) and (11) for the  
 141 partial time and spaces derivative of the velocity potential are still valid in the vicinity  
 142 of the boundary, provided that  $x$ ,  $y$  and  $z$  are now considered as local coordinates at the

143 boundary, the two first ones being parallel to the boundary and the  $z$  one perpendicular  
 144 to it. However, they do not simplify into eq. (12) at the boundary, even though  $k_x = k'_x$ ,  
 145  $k_y = k'_y$ , and  $k'_z = -k_z$ , because  $r' \neq r$ .

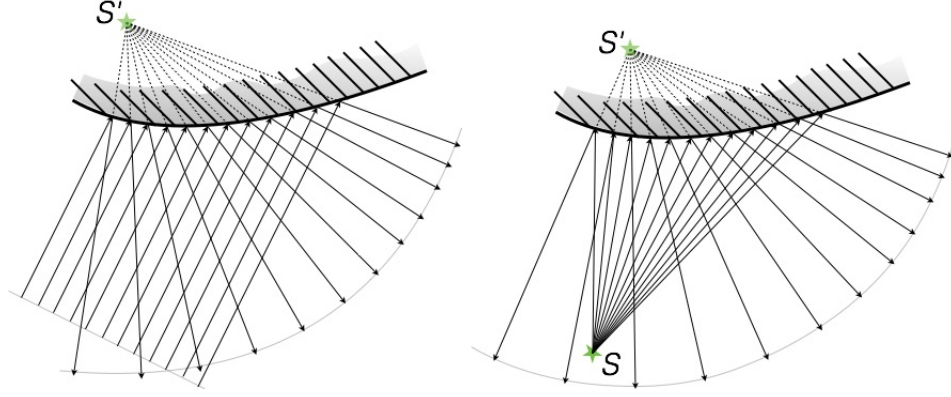


FIG. 4. Reflection on curved boundary. Left: plane wave impinging; right: source at finite distance. S is real source, and S' virtual image source.

146 On the boundary, we now obtain from eqs. (10) and (11):

$$\begin{aligned}\partial_x \psi &= -\frac{k_x}{\omega} \left( 1 + i \frac{\frac{\exp ikr}{kr^2} + R \frac{\exp ikr'}{kr'^2}}{\frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'}} \right) \partial_t \psi \\ \partial_y \psi &= -\frac{k_y}{\omega} \left( 1 + i \frac{\frac{\exp ikr}{kr^2} + R \frac{\exp ikr'}{kr'^2}}{\frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'}} \right) \partial_t \psi \\ \partial_z \psi &= -\frac{k_z}{\omega} \left( \frac{\left[ \frac{\exp ikr}{r} - R \frac{\exp ikr'}{r'} \right] + i \left[ \frac{\exp ikr}{kr^2} - R \frac{\exp ikr'}{kr'^2} \right]}{\frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'}} \right) \partial_t \psi = -\frac{\beta}{c} \partial_t \psi\end{aligned}$$

147 where

$$\frac{\left[ \frac{\exp ikr}{kr^2} + R \frac{\exp ikr'}{kr'^2} \right]}{\left[ \frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'} \right]} = \frac{\left[ \frac{1}{kr^2} + \frac{R'}{kr'^2} \right]}{\left[ \frac{1}{r} + \frac{R'}{r'} \right]} = \frac{1}{k} \frac{\frac{1}{r^3} + \left( \frac{1}{r} + \frac{1}{r'} \right) \frac{\Re(R')}{rr'} + \frac{|R'|^2}{r'^3} - i \left( \frac{1}{r} - \frac{1}{r'} \right) \frac{\Im(R')}{rr'}}{\frac{1}{r^2} + \frac{2\Re(R')}{rr'} + \frac{|R'|^2}{r'^2}}$$

148 with

$$\begin{aligned}
 R' = R \exp ik(r' - r) &= \frac{r' [\cos \theta (1 + \frac{i}{kr'}) - \beta]}{r [\cos \theta (1 + \frac{i}{kr'}) + \beta]} \\
 &= \frac{r' [\cos^2 \theta (1 + \frac{1}{k^2 r r'}) + \cos \theta \frac{\Im \beta}{k} (\frac{1}{r} - \frac{1}{r'}) - |\beta|^2]}{r [\cos^2 \theta (1 + \frac{1}{k^2 r'^2}) + 2 \cos \theta (\Re \beta + \frac{\Im \beta}{kr'}) + |\beta|^2]} \\
 &\quad + i \frac{r' \cos \theta [\frac{\cos \theta}{k} (\frac{1}{r} - \frac{1}{r'}) - 2 \Im \beta + \frac{\Re \beta}{k} (\frac{1}{r} + \frac{1}{r'})]}{r [\cos^2 \theta (1 + \frac{1}{k^2 r'^2}) + 2 \cos \theta (\Re \beta + \frac{\Im \beta}{kr'}) + |\beta|^2]} \quad (13)
 \end{aligned}$$

149 We then obtain:

$$\tan \phi_x = \tan \phi_y = \frac{1}{k} \frac{\frac{1}{r^3} + (\frac{1}{r} + \frac{1}{r'}) \frac{\Re(R')}{rr'} + \frac{|R'|^2}{r'^3}}{\frac{1}{r^2} + \frac{2\Re(R')}{rr'} + \frac{|R'|^2}{r'^2} + (\frac{1}{r} - \frac{1}{r'}) \frac{\Im(R')}{krr'}} \quad (14)$$

150 As a consequence, the phase angles  $\phi_x$  and  $\phi_y$  between the space derivatives of the velocity  
 151 potential with respect to  $x$  and  $y$  and its time derivative are position dependent - and depend  
 152 on the local angle of incidence  $\theta$  - through the modified reflection coefficient  $R'$ . No simple  
 153 equation subsists for eqs. (6) to reduce to.

154 Note that, in the case of a plane wave incident on a curved boundary (left pane of Fig.  
 155 4), recalculating eqs. (13) and (14) gives the simpler formulas:

$$\begin{aligned}
 R' &= \frac{r' [\cos \theta - \beta]}{\cos \theta (1 + \frac{i}{kr'}) + \beta} = r' \frac{[\cos^2 \theta - \cos \theta \frac{\Im \beta}{kr'} - |\beta|^2] - i \cos \theta [\frac{\cos \theta}{kr'} + 2 \Im \beta - \frac{\Re \beta}{kr'}]}{\cos^2 \theta (1 + \frac{1}{k^2 r'^2}) + 2 \cos \theta (\Re \beta + \frac{\Im \beta}{kr'}) + |\beta|^2} \\
 \tan \phi_x = \tan \phi_y &= \frac{1}{kr'^2} \frac{\Re(R') + \frac{|R'|^2}{r'}}{1 + \frac{2\Re(R')}{r'} + \frac{|R'|^2}{r'^2} - \frac{\Im(R')}{kr'^2}}
 \end{aligned}$$

## 156 VI. DISCUSSION

### 157 A. Absorption coefficient

158 The form for the absorption coefficient given by eq. (5) does not reduce to the form  
 159 given by Morse and Ingard (Morse and Ingard, 1968, p. 580) and introduced in acoustics

160 by Paris (Paris, 1927). However, but for a constant multiplicative factor, it is similar to  
 161 an expression given by Bosquet (Bosquet, 1967). As Bosquet further explains, the form  
 162 taken by the absorption coefficient strongly depends on the weighting function, that is, the  
 163 statistics one chooses for computing the mean absorption coefficient. In the present case,  
 164 the obvious choice is the so-called star-type statistics introduced by London (London, 1950),  
 165 that is, taking the mean value of the equation  $E_{tz} = \alpha E_{tt}$ :

$$\int_0^{\frac{\pi}{2}} E_{tz} \sin \theta d\theta = \alpha^* \int_0^{\frac{\pi}{2}} E_{tt} \sin \theta d\theta \quad (15)$$

166 Compared to the more usual bar-type statistics (Bosquet, 1967; London, 1950), the star-type  
 167 statistics does not include the factor  $\cos \theta$ ; but there is no need to introduce this factor as  
 168  $E_{tz}$  already contains it, since it is the normal flux into the wall. Therefore is the star-type  
 169 statistics the obvious choice. Assuming as usual an isotropic distribution of wave directions,  
 170 the term  $|\partial_t \psi|^2$  is then constant for all wave directions in the first line of eq. (3) and in eq.  
 171 (4). The left hand side of eq. (15) simply reduces to the mean value of  $\sin \theta$ ; and the right  
 172 hand side also includes the mean value of  $\sin^3 \theta$ . Therefore, one obtains:

$$\alpha^* = \frac{2 \Re(\beta)}{1 + \frac{2}{3} + |\beta|^2} \quad (16)$$

173 which is the value of  $\alpha(\theta)$  taken for  $\theta = 55^\circ$ , an angle that has repeatedly appeared since  
 174 the 1950s as the incidence angle that represents the mean absorption (Bosquet, 1967; Guig-  
 175 nouard, 1991; Vogel, 1956). Indeed,  $\sin^2 55^\circ = 0.67 \approx \frac{2}{3}$ .

176 Simple calculation shows that the absorption coefficients given by eq. (5) and (16) take  
 177 values between 0 and a maximum reached when  $\Re(\beta) = \sqrt{1 + \sin^2 \theta + \Im^2(\beta)}$  and  $\Re(\beta) =$   
 178  $\sqrt{1 + \frac{2}{3} + \Im^2(\beta)}$  respectively. This maximum is respectively equal to  $\frac{1}{\sqrt{1 + \sin^2 \theta + \Im^2(\beta)}}$  and



179  $\frac{1}{\sqrt{1+\frac{2}{3}+\Im^2(\beta)}}$ , that is, reaches its absolute maximum for  $\Im(\beta) = 0$  with value  $\frac{1}{\sqrt{1+\sin^2 \theta}}$  and  
 180  $\frac{1}{\sqrt{1+\frac{2}{3}}} \approx 0.775$  respectively. Both absolute maxima are therefore smaller than 1, except at  
 181 normal incidence ( $\theta = 0$ ) for the first one.

182 Last but not least, integrating eq. (16) on octave or third octave energy bands leads to  
 183 the usual definition of band related absorption coefficients.

## 184 B. Scattering coefficients

185 The scattering coefficients given by eq. (6) are independent of the incidence angle  $\theta$ ,  
 186 except through the reflection coefficient in the expression of  $\tan \phi_x$  and  $\tan \phi_y$  for curved  
 187 boundaries. However, the real part of the wall admittance contributes to the scattering  
 188 coefficients. In other words, absorption on the wall induces scattering. This is a novel  
 189 result.

190 Also novel is the observation that scattering coefficients are not bounded: they can take  
 191 any value between  $-\infty$  and  $+\infty$ , as is easily seen from eq. (6).

192 Traditionally, scattering is considered as the result of two mechanisms (Embrecchts, 2002):  
 193 finite dimensions of reflecting surfaces; and roughness of the reflecting surfaces. There is no  
 194 doubt from Sect. VC that the second term of eq. (6) corresponds to scattering by surface  
 195 roughness, since it is driven by the finite distance from the surface to the image source.  
 196 However, this term subsists for sources located at finite distances from a flat reflecting  
 197 surface (see Sect. VB), as long as its admittance admits an imaginary part. But roughness  
 198 is usually associated with locally varying imaginary parts of the admittance, as can be easily

199 demonstrated with Schroeder diffusers, where the depths of the wells drive this imaginary  
 200 part. In the present case, no locally varying admittance is necessary to induce scattering.

201 We therefore suspect that the mechanism described in Sect. V does not correspond  
 202 to traditional scattering, but to a different effect. Indeed, in the work of Dujourdy *et al.*  
 203 (Dujourdy *et al.*, 2017, 2019) and Meacham *et al.* (Meacham *et al.*, 2019), the scattering  
 204 coefficient induces a decrease of steady-state energy with distance. Although scattering does  
 205 induce a decrease of steady-state energy with distance by redirecting part of it toward the  
 206 source, no redirection is here at stake, just a reduction of the acoustical intensity by friction  
 207 on the walls. In other words, total energy and acoustical intensity can be considered as  
 208 two coupled systems that each follow its own conservation equation with some exchange  
 209 between the two systems. As is well known from such coupled systems, the less damped  
 210 system drives the energy decay with time, which precisely is what is obtained in Fig. 1.

211 As a consequence, the scattering coefficient  $D$  is ill-named. A better designation would  
 212 be *intensity-friction* coefficient, or simply *friction* coefficient. For the time being, we stick  
 213 to usage and keep the designation "scattering coefficient".

214 Note that, as for the absorption coefficient, integrating eq. (6) on octave or third octave  
 215 energy bands leads to band related scattering - or friction - coefficients.

### 216 C. Admittance, impedance, and acoustic resistance

217 From the expression of the absorption and scattering coefficients (eqs. 5 and 6), it becomes  
 218 possible to express the specific acoustic admittance in the case of flat walls, which is the case

219 of Fig. 1. From this expression, the specific acoustic impedance and the acoustic resistance  
 220 of the wall can also be computed.

221 Table I presents the value of the octave-band absorption and scattering coefficients of  
 222 the hallway of Fig. 1, derived from the figure itself by taking the centre of the vertical  
 223 green patches. There is no green patch in Fig. 1 for the 125 Hz octave band, but the two  
 224 stripes are tangent to each other at the point used in Table I. Note that, according to eq.  
 225 (6), the real part of the specific acoustic admittance is equal to the scattering coefficient  
 226 (plane walls). The imaginary part is obtained from eq. (5) which gives valid values for all  
 227 octave bands. Two possibilities exist here: either considering a diffuse sound field, and use  
 228 the average absorption coefficient of eq. (16); or considering that the sound field remains  
 229 parallel to the walls, and use eq. (5) with  $\sin \theta = 0$ . The latter case gave more consistent  
 230 results.

231 The specific acoustic impedance is then computed as the inverse of the specific acoustic  
 232 admittance, and multiplying its real part by the characteristic impedance of air  $\rho_0 c = 410$   
 233  $\text{Ns/m}^3$  approximates the acoustic resistance of the wall - also in  $\text{Ns/m}^3$ . No attempt has  
 234 been made to compare the obtained values to typical walls, given that they are made of  
 235 particle boards and gyps boards, with a hard plastic floor. Indeed, quick evaluation of  
 236 the resonances of the walls by knocking on them revealed a complex structure with many  
 237 resonances spread over all the octave bands of interest, rendering inconsistent the parsing  
 238 of the measured impedance into resonances.

TABLE I. Admittance, impedance and acoustic resistance of the walls of the hallway of Fig. 1.

octave band	63Hz	125Hz	250Hz	500Hz	1kHz	2kHz	4kHz	8kHz
absorption coeff.	0.025	0.115	0.1	0.095	0.09	0.12	0.14	0.13
scattering coeff.	0.6	0.125	0.335	0.355	0.22	0.135	0.145	0.23
$\Re(\beta)$	0.6	0.125	0.335	0.355	0.22	0.135	0.145	0.23
$\Im(\beta)$	6.83	1.08	2.036	2.52	1.96	1.11	1.02	1.58
$\Re(\zeta)$	0.013	0.106	0.059	0.055	0.057	0.108	0.135	0.091
$\Im(\zeta)$	-0.15	-0.92	-0.41	-0.39	-0.50	-0.89	-0.96	-0.62
resistance ( $Ns/m^3$ )	5.23	43.7	24.1	22.5	23.2	44.3	55.5	37.1

## 239 VII. CONCLUSION

240 Starting with the precedent papers of Dujourdy *et al.* (Dujourdy *et al.*, 2017, 2019)  
241 that have shown that integrating the stress-energy tensor in a disproportionate enclosure  
242 and taking into account the boundary conditions leads to "loss terms" that correspond to  
243 absorption and scattering, we expressed the elements of the stress-energy tensor with the  
244 help of the wall admittance relation between normal derivative and time derivative of the  
245 velocity potential on the wall.

246 As expected, the absorption coefficients are proportional to the real part of the wall  
247 admittance. However, they also depend on the angle of incidence of the wave, leading to an  
248 expression of the absorption coefficient eq. (16) which has been signaled in the literature,

249 for example by Bosquet (Bosquet, 1967), yet is not very usual; and to scattering coefficients  
250 additively involving the real and imaginary parts of the wall admittance, as seen in eq.  
251 (6), proving the absorptive nature of the scattering coefficient in the stress-energy tensor  
252 formalism.

253 In the case of plane boundaries, calculations can be extended to evaluate the mean  
254 impedance of the walls. An example is given for a long hallway.

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## 256 AUTHOR DECLARATIONS

### 257 Conflict of Interest

258 The authors have no conflicts to disclose.

### 259 Data Availability

260 Data sharing not applicable – no new data generated.

261

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