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## Absorptive nature of scattering coefficients in stress-energy tensor formalism for room acoustics

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(Dated: 21 February 2024)

1 In the stress-energy tensor formalism, the symmetry between absorption and scatter-  
 2 ing coefficients, as proven by measurements combined with simulations, is counter-  
 3 intuitive. By introducing the wall admittance, we show that the scattering coefficient  
 4 is partly created by the real part of the wall admittance combined with the active  
 5 intensity, that is, is partly due to absorption. However, for curved surfaces or finite  
 6 source distances, it also depends on the imaginary part of the wall admittance in  
 7 combination with the reactive intensity, which confers it genuine scattering proper-  
 8 ties inversely proportional to the distances to the sources. Thus, for plane waves  
 9 impinging on plane boundaries, or purely real admittances, scattering reduces to  
 10 absorption.

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## 11 I. INTRODUCTION

12 (Added: The stress-energy tensor formalism was initially introduced by Morse and col-  
 13 laborators (Morse and Feshbach, 1956; Morse and Ingard, 1968) in order to describe energy  
 14 conservation in linear acoustics. Many years later, Stanzial and collaborators applied the  
 15 conservation of the stress-energy tensor to room acoustics (Stanzial *et al.*, 2002), focusing  
 16 on the expression of the radiation pressure. In a subsequent paper (Stanzial and Schiffrer,  
 17 2010), they introduced the energy velocity as ratio of the sound intensity to the total acous-  
 18 tic energy, and derived from the conservation of the stress-energy tensor an expression for  
 19 the local initial reverberation time at the onset of energy decay. Some related ideas, link-  
 20 ing wave impedance to the complex sound intensity, are also found in (Mann *et al.*, 1987;  
 21 Stanzial and Graffigna, 2017). Mann *et al.* focused on energy transfer and analysed in terms  
 22 of energy flux and power what happens in the vicinity of intensity vortex that are created  
 23 by specific configurations of sources. Stanzial and Graffigna went one step further in the  
 24 definition and measurement of complex intensity. However, both papers are outside the  
 25 scope of the present analysis that focuses on the active stress-energy tensor - see (Polack,  
 26 2023) for an extension to the *complex* stress-energy tensor and its conservation, together  
 27 with preliminary measurement results.)

28 (Replaced: ~~The stress-energy tensor formalism~~ replaced with: A somewhat different per-  
 29 spective) was introduced by Dujourdy *et al.* (Dujourdy *et al.*, 2017, 2019) (Added: in order)  
 30 to generalize the diffusion equation formalism of Ollendorff and Picaut (Ollendorff, 1969;  
 31 Picaut *et al.*, 1997). Indeed, where the diffusion equation arbitrarily introduces a gradi-

ent type relationship between sound intensity and total energy, the stress-energy tensor formalism introduces instead the conservation equation for intensity. When integrated in disproportionate enclosures, the stress-energy tensor yields both absorption and scattering on the boundaries. Dujourdy *et al.* stressed the symmetrical role played by absorption and scattering coefficients in the conservation equations, which was then systematically investigated and confirmed by Meacham *et al.* (Meacham *et al.*, 2019). The present papers aims at proving this symmetry and the absorbing nature of the scattering coefficient, using the local admittance formalism at the boundaries.

After presenting the background of the stress-energy conservation, we successively express the elements of the stress energy tensor on the boundaries, then the absorption and scattering coefficients, in terms of the local admittance. We then examine the main types of boundaries, and discuss the relationship between absorption and scattering coefficients.

## II. THE BASIC EQUATIONS

Dujourdy *et al.* (Dujourdy *et al.*, 2017, 2019) have shown that the wave equation, satisfied by the velocity potential, can be extended by a set of conservation equations that reduces to the conservation of the stress-energy tensor  $\underline{T}$ .

Let  $\psi$  be the velocity potential. The conservation of the stress-energy tensor is expressed as:

$$\vec{\nabla} \cdot \underline{T} = 0 \tag{1}$$

50 with

$$\underline{\underline{T}} = \begin{pmatrix} E_{tt} & E_{tx} & E_{ty} & E_{tz} \\ E_{tx} & E_{xx} & E_{xy} & E_{xz} \\ E_{ty} & E_{xy} & E_{yy} & E_{yz} \\ E_{tz} & E_{xz} & E_{yz} & E_{zz} \end{pmatrix}$$

In terms of the velocity potential, the elements of the stress-energy tensor are given by:

$$E_{tt} = \frac{\rho}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\vec{\nabla} \psi|^2 \right)$$

$$E_{tx} = -\frac{\rho_0}{c} \partial_t \psi \partial_x \psi$$

$$E_{ty} = -\frac{\rho_0}{c} \partial_t \psi \partial_y \psi$$

$$E_{tz} = -\frac{\rho_0}{c} \partial_t \psi \partial_z \psi$$

$$E_{xx} = \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\partial_x \psi|^2 - |\partial_y \psi|^2 - |\partial_z \psi|^2 \right)$$

$$E_{yy} = \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 - |\partial_x \psi|^2 + |\partial_y \psi|^2 - |\partial_z \psi|^2 \right)$$

$$E_{zz} = \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 - |\partial_x \psi|^2 - |\partial_y \psi|^2 + |\partial_z \psi|^2 \right)$$

$$E_{xy} = \rho_0 \partial_x \psi \partial_y \psi$$

$$E_{xz} = \rho_0 \partial_x \psi \partial_z \psi$$

$$E_{yz} = \rho_0 \partial_y \psi \partial_z \psi$$

51 where  $\rho_0$  is the (Added: steady-state) density of air and  $c$  the speed of sound.

52 In the specific cases of long enclosures (Dujourdy *et al.*, 2017) and flat spaces (Dujourdy  
53 *et al.*, 2019), Dujourdy *et al.* were able to reduce the conservation of the stress-energy tensor

54 (eq. 1) to the telegraphers equation:

$$\frac{1}{c^2}\partial_{tt}E - \partial_{xx}E + \frac{A+D}{\lambda c}\partial_tE + \frac{AD}{\lambda^2}E = 0 \quad (2)$$

55 where  $E$  is the total energy,  $\lambda$  is the mean free path of the enclosure, and  $A$  and  $D$  are  
 56 respectively modified absorption and scattering coefficients at the boundaries. Eq. (2) is  
 57 symmetrical with respect to the absorption and scattering coefficients, as was confirmed by  
 58 Meacham *et al.* (Meacham *et al.*, 2019) who systematically explored the whole space of values  
 59 for the absorption and scattering coefficients and compared thus obtained reverberation  
 60 times and spatial decays with the measured ones in a hallway. As shown in Fig. 1, the  
 61 measured reverberation times and spatial decays correspond to narrow stripes of values for  
 62  $A$  and  $D$ , which do not always overlap; but most striking in Fig. 1 is the symmetry of the  
 63 figure with respect to absorption and scattering coefficients. Physical justification of this  
 64 symmetry is therefore the goal of the present paper.

### 65 III. WALL ADMITTANCE

66 Now, wall admittance introduces a relation between the normal derivative and the time  
 67 derivative of the velocity potential on the wall:

$$-\partial_n\psi = \frac{\beta}{c}\partial_t\psi = \frac{1}{c\zeta}\partial_t\psi$$

with  $\vec{n}$  the exterior normal to the wall,  $\zeta$  the (Replaced: ~~reduced~~ replaced with: **specific acoustic**) impedance of the wall (Added: (Morse and Ingard, 1968, p. 580)), (Added: **that is, the wall impedance normalized by the characteristic impedance  $\rho_0c$  of air,**) and  $\beta = \frac{1}{\zeta}$

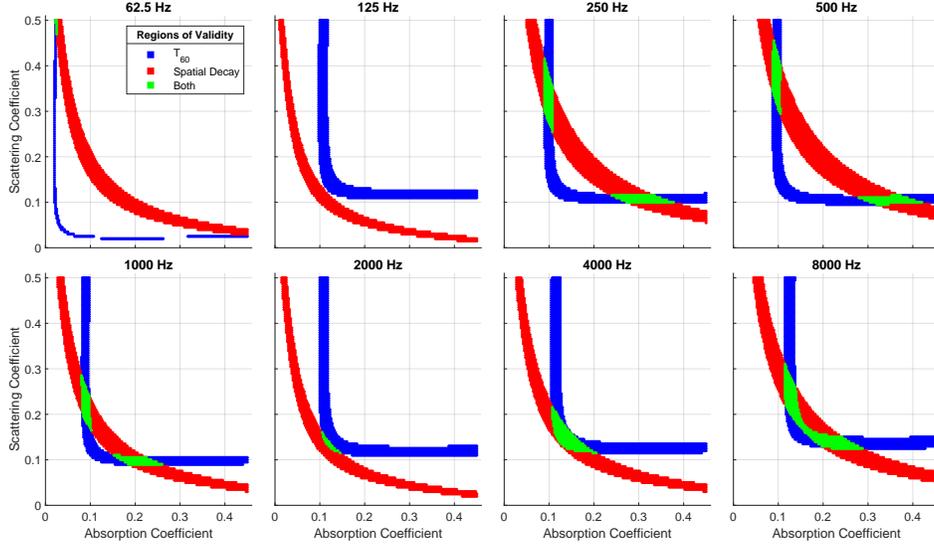


FIG. 1. Relevant combinations of absorption and scattering coefficients leading to measured reverberation times and spatial decays in a hallway (reproduced from (Meacham *et al.*, 2019)).

(Added: is) the (Replaced: ~~reduced~~ replaced with: *specific acoustic*) admittance. Let us then consider a flat enclosure with vertical coordinate  $z$ . We can express the elements  $E_z$  of the stress-energy tensor, where  $\cdot$  takes the values  $t$ ,  $x$ , or  $y$ . However, the admittance formalism implies a complex velocity potential, whereas the elements of the stress-energy tensor must remain real, as is expected from energy quantities. Thus, the expression for the

non-diagonal terms of the stress-energy tensor must be modified as:

$$E_{tx} = -\frac{\rho_0}{c} \Re(\partial_t \psi \partial_x \psi^*)$$

$$E_{ty} = -\frac{\rho_0}{c} \Re(\partial_t \psi \partial_y \psi^*)$$

$$E_{tz} = -\frac{\rho_0}{c} \Re(\partial_t \psi \partial_z \psi^*)$$

$$E_{xy} = \rho_0 \Re(\partial_x \psi \partial_y \psi^*)$$

$$E_{xz} = \rho_0 \Re(\partial_x \psi \partial_z \psi^*)$$

$$E_{yz} = \rho_0 \Re(\partial_y \psi \partial_z \psi^*)$$

68 where the exponent \* means complex conjugated.

69 We thus obtain on the "ceiling":

$$\begin{aligned} E_{tz} &= \frac{\rho_0}{c^2} \Re(\partial_t \psi \beta^* \partial_t \psi^*) = \frac{\rho_0}{c^2} \Re(\beta) |\partial_t \psi|^2 \\ E_{xz} &= -\frac{\rho_0}{c} \Re(\partial_x \psi \beta^* \partial_t \psi^*) = \Re(\beta) E_{tx} + \Im(\beta) \Im\left(\frac{\rho_0}{c} \partial_x \psi \partial_t \psi^*\right) \\ E_{yz} &= -\frac{\rho_0}{c} \Re(\partial_y \psi \beta^* \partial_t \psi^*) = \Re(\beta) E_{ty} + \Im(\beta) \Im\left(\frac{\rho_0}{c} \partial_y \psi \partial_t \psi^*\right) \end{aligned} \quad (3)$$

70 and the same equations with opposite sign on the "floor". (Added: Note that  $E_{xz}$  and  $E_{yz}$   
71 correspond to the parallel components of the radiation pressure exerted on the "ceiling" and  
72 the "floor" (Stanzial *et al.*, 2002).)

#### 73 IV. INTERPRETATION

74 Recalling that  $E_{tx}$  and  $E_{ty}$  are the components of the sound intensity in the  $x$  and  
75  $y$  directions, we observe that the boundary conditions for  $E_{xz}$  and  $E_{yz}$  involve not only  
76 the active intensity associated with the real part of the admittance, but also the reactive

77 intensity associated with the imaginary part of the admittance. This is not surprising, as  
 78 the reactive intensity corresponds to local recirculation of energy, just as the imaginary part  
 79 of the wall admittance is associated with a non-flat surface. Both phenomena contribute to  
 80 the scattering of the impinging waves.

However, recalling that admittances are defined for pure waves only, we can further sim-  
 plify these expressions by considering the phase angles  $\phi_x$  and  $\phi_y$  between the space deriva-  
 tives of the velocity potential with respect to  $x$  and  $y$  respectively, and its time derivative.  
 The last two ceiling equations then reduce to:

$$E_{xz} = [\Re(\beta) + \Im(\beta) \tan \phi_x] E_{tx}$$

$$E_{yz} = [\Re(\beta) + \Im(\beta) \tan \phi_y] E_{ty}$$

As for  $E_{tz}$ , we must first express  $E_{tt}$  as a function of the wall admittance, that is:

$$\begin{aligned}
 E_{tt} &= \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\vec{\nabla} \psi|^2 \right) \\
 &= \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\partial_x \psi|^2 + |\partial_y \psi|^2 + |\partial_z \psi|^2 \right) \\
 &= \frac{\rho_0}{2} \left( \frac{1}{c^2} |\partial_t \psi|^2 + |\partial_x \psi|^2 + |\partial_y \psi|^2 + \left| \frac{\beta}{c} \partial_t \psi \right|^2 \right) \\
 &= \frac{\rho_0}{2} \left( \frac{1}{c^2} [1 + |\beta|^2] |\partial_t \psi|^2 + |\partial_x \psi|^2 + |\partial_y \psi|^2 \right)
 \end{aligned}$$

81 Introducing the angle of incidence  $\theta$  of the wave makes it possible to express the last two  
 82 terms in the previous equation as:

$$|\partial_x \psi|^2 + |\partial_y \psi|^2 = \frac{\sin^2 \theta}{c^2} |\partial_t \psi|^2$$

83 that is:

$$E_{tt} = \frac{\rho_0}{2c^2} (1 + \sin^2 \theta + |\beta|^2) |\partial_t \psi|^2 \quad (4)$$

84 leading to the following expression for the absorption coefficient:

$$\alpha(\theta) = \frac{E_{tz}}{E_{tt}} = \frac{2 \Re(\beta)}{1 + \sin^2 \theta + |\beta|^2} \quad (5)$$

85 The interpretations of the scattering coefficients  $D_x$  and  $D_y$  are similar. We simply  
86 obtain:

$$D_x = \frac{E_{xz}}{E_{tx}} = [\Re(\beta) + \Im(\beta) \tan \phi_x] \quad (6)$$

$$D_y = \frac{E_{yz}}{E_{ty}} = [\Re(\beta) + \Im(\beta) \tan \phi_y]$$

## 87 V. BOUNDARY CONDITIONS

88 The evaluation of eqs. (6) takes different forms depending on the type of boundary and  
89 the position of the source.

### 90 A. Plane wave on flat boundary

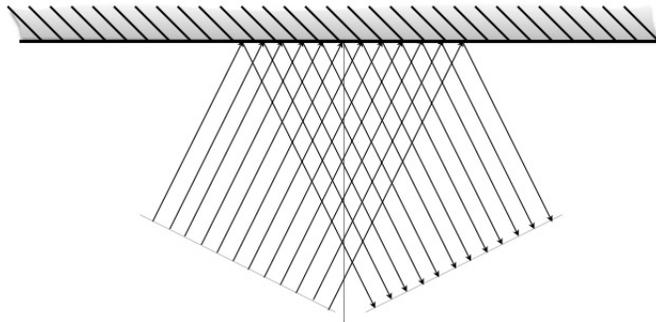


FIG. 2. Plane wave reflecting on flat boundary.

91 For a plane wave impinging on a flat boundary  $z = 0$  (Fig. 2), the velocity potential  
92 takes the following form:

$$\psi(t, r) = \exp -i(\omega t - k_x x - k_y y - k_z z) + R \exp -i(\omega t - k_x x - k_y y + k_z z) \quad (7)$$

where  $\omega$  is the radian frequency,  $k_x$ ,  $k_y$  and  $k_z$  the components of the wave number  $k = \frac{\omega}{c}$  along the three coordinates, and  $R$  the complex reflection coefficient on the boundary. The first term in eq. (7) represents the impinging wave; and the second the reflected wave. The time derivative in the neighbourhood of the boundary is given by:

$$\partial_t \psi = -i\omega \psi = -i\omega \exp -i(\omega t - k_x x - k_y y) [\exp ik_z z + R \exp -ik_z z]$$

93 and the space derivatives by:

$$\partial_x \psi = ik_x \psi = ik_x \exp -i(\omega t - k_x x - k_y y) [\exp ik_z z + R \exp -ik_z z] = -\frac{k_x}{\omega} \partial_t \psi$$

$$\partial_y \psi = ik_y \psi = ik_y \exp -i(\omega t - k_x x - k_y y) [\exp ik_z z + R \exp -ik_z z] = -\frac{k_y}{\omega} \partial_t \psi \quad (8)$$

$$\partial_z \psi = ik_z \exp -i(\omega t - k_x x - k_y y) [\exp ik_z z - R \exp -ik_z z] = -\frac{\beta}{c} \partial_t \psi$$

As a consequence, on the boundary  $z = 0$ , the phase angles  $\phi_x$  and  $\phi_y$  between the space derivatives of the velocity potential with respect to  $x$  and  $y$  and its time derivative are both equal to 0. In this case, eqs. (6) simply reduce to:

$$D_x = D_y = \Re(\beta)$$

94 that is, the scattering coefficient is purely created by absorption. Note that the last line  
 95 of eq. (8), together with the relation between the normal component of the wave number  
 96 and the angle of incidence  $k_z = k \cos \theta = \frac{\omega}{c} \cos \theta$ , gives the expected relation between the  
 97 complex reflection coefficient and the admittance:

$$R = \frac{\cos \theta - \beta}{\cos \theta + \beta} = \frac{[\cos^2 \theta - |\beta|^2] - 2i \cos \theta \Im \beta}{\cos^2 \theta + 2 \cos \theta \Re \beta + |\beta|^2}$$

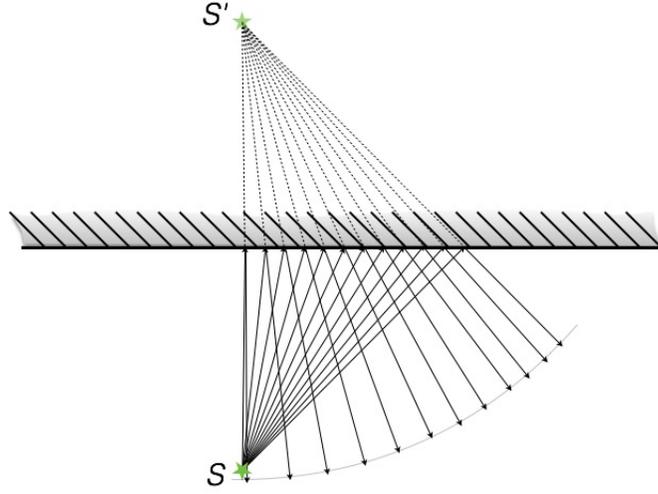


FIG. 3. Source at finite distance from flat boundary;  $S$  is real source, and  $S'$  virtual image source.

98 **B. Source at finite distance from flat boundary**

99 For a spherical wave impinging on a flat boundary (Fig. 3), the velocity potential takes  
 100 a different form:

$$\psi(t, r) = \frac{\exp -i(\omega t - kr)}{4\pi r} + R \frac{\exp -i(\omega t - kr')}{4\pi r'} \quad (9)$$

101 where  $R$  still is the complex reflection coefficient on the boundary,  $r$  the distance to the real  
 102 source  $S$ , and  $r'$  the distance to the virtual image source  $S'$  (see Fig. 3). The first term in eq.  
 103 (9) represents the impinging wave; and the second the reflected wave. The time derivative  
 104 in the neighbourhood of the boundary is now given by:

$$\partial_t \psi = -i\omega \psi = -i\omega \frac{\exp -i\omega t}{4\pi} \left[ \frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'} \right] \quad (10)$$

105 and the space derivatives by:

$$\begin{aligned}
 \partial_x \psi &= i \frac{\exp -i\omega t}{4\pi} \left[ k_x \frac{\exp ikr}{r} \left(1 + \frac{i}{kr}\right) + R k'_x \frac{\exp ikr'}{r'} \left(1 + \frac{i}{kr'}\right) \right] \\
 \partial_y \psi &= i \frac{\exp -i\omega t}{4\pi} \left[ k_y \frac{\exp ikr}{r} \left(1 + \frac{i}{kr}\right) + R k'_y \frac{\exp ikr'}{r'} \left(1 + \frac{i}{kr'}\right) \right] \\
 \partial_z \psi &= i \frac{\exp -i\omega t}{4\pi} \left[ k_z \frac{\exp ikr}{r} \left(1 + \frac{i}{kr}\right) + R k'_z \frac{\exp ikr'}{r'} \left(1 + \frac{i}{kr'}\right) \right]
 \end{aligned} \tag{11}$$

106 where  $k_x$ ,  $k_y$  and  $k_z$  are the components of the incident wave number  $k$  along the three  
 107 coordinates, and  $k'_x$ ,  $k'_y$ , and  $k'_z$  those of the reflected wave number  $k'$ . At the boundary, the  
 108 distance to the real and imaginary sources are equal, therefore  $r = r'$ ; and  $k_x = k'_x$ ,  $k_y = k'_y$ ,  
 109 but  $k'_z = -k_z$ . The partial derivatives further reduce to:

$$\begin{aligned}
 \partial_t \psi &= -i\omega \frac{\exp -i(\omega t - kr)}{4\pi r} (1 + R) \\
 \partial_x \psi &= ik_x \frac{\exp -i(\omega t - kr)}{4\pi r} (1 + R) \left(1 + \frac{i}{kr}\right) = -\frac{k_x}{\omega} \left(1 + \frac{i}{kr}\right) \partial_t \psi \\
 \partial_y \psi &= ik_y \frac{\exp -i(\omega t - kr)}{4\pi r} (1 + R) \left(1 + \frac{i}{kr}\right) = -\frac{k_y}{\omega} \left(1 + \frac{i}{kr}\right) \partial_t \psi \\
 \partial_z \psi &= ik_z \frac{\exp -i(\omega t - kr)}{4\pi r} (1 - R) \left(1 + \frac{i}{kr}\right) = -\frac{\beta}{c} \partial_t \psi
 \end{aligned} \tag{12}$$

As a consequence, on the boundary, the phase angles  $\phi_x$  and  $\phi_y$  between the space derivatives  
 of the velocity potential with respect to  $x$  and  $y$  and its time derivative are both equal to  
 arctan  $\frac{1}{kr}$ . And eqs. (6) reduce to:

$$D_x = D_y = \Re(\beta) + \frac{\Im(\beta)}{kr}$$

110 In this case, the imaginary part of the admittance contributes to the scattering coefficient,  
 111 and this contribution is inversely proportional to  $kr$ . And according to the last line of eq.  
 112 (12), the usual relation between the complex reflection coefficient and the admittance is

113 modified into:

$$R = \frac{\cos \theta \left(1 + \frac{i}{kr}\right) - \beta}{\cos \theta \left(1 + \frac{i}{kr}\right) + \beta} = \frac{[\cos^2 \theta \left(1 + \frac{1}{k^2 r^2}\right) - |\beta|^2] - 2i \cos \theta \left[\Im \beta + \frac{\Re \beta}{kr}\right]}{\cos^2 \theta \left(1 + \frac{1}{k^2 r^2}\right) + 2 \cos \theta \left(\Re \beta + \frac{\Im \beta}{kr}\right) + |\beta|^2}$$

114 Note that this expression is equivalent to introducing a complex angle of incidence  $\theta$ .

### 115 C. Curved boundary

116 In the case of a curved boundary, the real and virtual sources are not located at the  
 117 same distance from the boundary (Fig. 4). As a consequence, eqs. (10) and (11) for the  
 118 partial time and spaces derivative of the velocity potential are still valid in the vicinity  
 119 of the boundary, provided that  $x$ ,  $y$  and  $z$  are now considered as local coordinates at the  
 120 boundary, the two first ones being parallel to the boundary and the  $z$  one perpendicular  
 121 to it. However, they do not simplify into eq. (12) at the boundary, even though  $k_x = k'_x$ ,  
 122  $k_y = k'_y$ , and  $k'_z = -k_z$ , because  $r' \neq r$ .

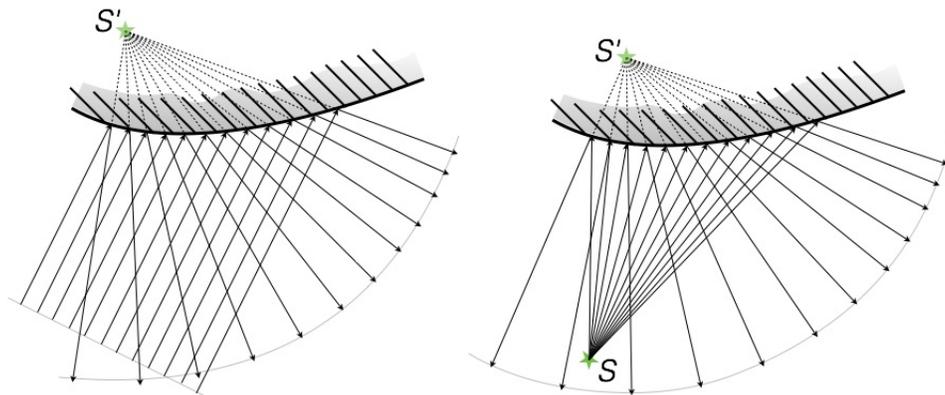


FIG. 4. Reflection on curved boundary. Left: plane wave impinging; right: source at finite distance. S is real source, and S' virtual image source.

On the boundary, we now obtain from eqs. (10) and (11):

$$\begin{aligned}\partial_x \psi &= -\frac{k_x}{\omega} \left( 1 + i \frac{\frac{\exp ikr}{kr^2} + R \frac{\exp ikr'}{kr'^2}}{\frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'}} \right) \partial_t \psi \\ \partial_y \psi &= -\frac{k_y}{\omega} \left( 1 + i \frac{\frac{\exp ikr}{kr^2} + R \frac{\exp ikr'}{kr'^2}}{\frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'}} \right) \partial_t \psi \\ \partial_z \psi &= -\frac{k_z}{\omega} \left( \frac{\left[ \frac{\exp ikr}{r} - R \frac{\exp ikr'}{r'} \right] + i \left[ \frac{\exp ikr}{kr^2} - R \frac{\exp ikr'}{kr'^2} \right]}{\frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'}} \right) \partial_t \psi = -\frac{\beta}{c} \partial_t \psi\end{aligned}$$

where

$$\frac{\left[ \frac{\exp ikr}{kr^2} + R \frac{\exp ikr'}{kr'^2} \right]}{\left[ \frac{\exp ikr}{r} + R \frac{\exp ikr'}{r'} \right]} = \frac{\left[ \frac{1}{kr^2} + \frac{R'}{kr'^2} \right]}{\left[ \frac{1}{r} + \frac{R'}{r'} \right]} = \frac{1}{k} \frac{\frac{1}{r^3} + \left( \frac{1}{r} + \frac{1}{r'} \right) \frac{\Re(R')}{rr'} + \frac{|R'|^2}{r'^3} - i \left( \frac{1}{r} - \frac{1}{r'} \right) \frac{\Im(R')}{rr'}}{\frac{1}{r^2} + \frac{2\Re(R')}{rr'} + \frac{|R'|^2}{r'^2}}$$

with

$$\begin{aligned}R' = R \exp ik(r' - r) &= \frac{r' \left[ \cos \theta \left( 1 + \frac{i}{kr} \right) - \beta \right]}{r \left[ \cos \theta \left( 1 + \frac{i}{kr'} \right) + \beta \right]} \\ &= \frac{r' \left[ \cos^2 \theta \left( 1 + \frac{1}{k^2 rr'} \right) + \cos \theta \frac{\Im \beta}{k} \left( \frac{1}{r} - \frac{1}{r'} \right) - |\beta|^2 \right]}{r \left[ \cos^2 \theta \left( 1 + \frac{1}{k^2 r'^2} \right) + 2 \cos \theta \left( \Re \beta + \frac{\Im \beta}{kr'} \right) + |\beta|^2 \right]} \\ &\quad + i \frac{r' \cos \theta \left[ \frac{\cos \theta}{k} \left( \frac{1}{r} - \frac{1}{r'} \right) - 2\Im \beta + \frac{\Re \beta}{k} \left( \frac{1}{r} + \frac{1}{r'} \right) \right]}{r \left[ \cos^2 \theta \left( 1 + \frac{1}{k^2 r'^2} \right) + 2 \cos \theta \left( \Re \beta + \frac{\Im \beta}{kr'} \right) + |\beta|^2 \right]}\end{aligned}\quad (13)$$

123 We then obtain:

$$\tan \phi_x = \tan \phi_y = \frac{1}{k} \frac{\frac{1}{r^3} + \left( \frac{1}{r} + \frac{1}{r'} \right) \frac{\Re(R')}{rr'} + \frac{|R'|^2}{r'^3}}{\frac{1}{r^2} + \frac{2\Re(R')}{rr'} + \frac{|R'|^2}{r'^2} + \left( \frac{1}{r} - \frac{1}{r'} \right) \frac{\Im(R')}{krr'}}\quad (14)$$

124 As a consequence, the phase angles  $\phi_x$  and  $\phi_y$  between the space derivatives of the velocity  
 125 potential with respect to  $x$  and  $y$  and its time derivative are position dependent - and depend  
 126 on the local angle of incidence  $\theta$  - through the modified reflection coefficient  $R'$ . No simple  
 127 equation subsists for eqs. (6) to reduce to.

Note that, in the case of a plane wave incident on a curved boundary (left pane of Fig. 4), recalculating eqs. (13) and (14) gives the simpler formulas:

$$R' = \frac{r' [\cos \theta - \beta]}{\cos \theta \left(1 + \frac{i}{kr'}\right) + \beta} = r' \frac{[\cos^2 \theta - \cos \theta \frac{\Im \beta}{kr'} - |\beta|^2] - i \cos \theta \left[\frac{\cos \theta}{kr'} + 2\Im \beta - \frac{\Re \beta}{kr'}\right]}{\cos^2 \theta \left(1 + \frac{1}{k^2 r'^2}\right) + 2 \cos \theta \left(\Re \beta + \frac{\Im \beta}{kr'}\right) + |\beta|^2}$$

$$\tan \phi_x = \tan \phi_y = \frac{1}{kr'^2} \frac{\Re(R') + \frac{|R'|^2}{r'}}{1 + \frac{2\Re(R')}{r'} + \frac{|R'|^2}{r'^2} - \frac{\Im(R')}{kr'^2}}$$

## 128 VI. DISCUSSION

### 129 A. Absorption coefficient

130 The form for the absorption coefficient given by eq. (5) does not reduce to the form  
 131 given by Morse and Ingard (Added: (Morse and Ingard, 1968, p. 580)) and introduced in  
 132 acoustics by Paris (Paris, 1927). However, but for a constant multiplicative factor, it is  
 133 similar to an expression given by Bosquet (Bosquet, 1967). As Bosquet further explains, the  
 134 form taken by the absorption coefficient strongly depends on the weighting function, that  
 135 is, the statistics one chooses for computing the mean absorption coefficient. In the present  
 136 case, the obvious choice is the so-called star-type statistics introduced by London (London,  
 137 1950), that is, taking the mean value of the equation  $E_{tz} = \alpha E_{tt}$ :

$$\int_0^{\frac{\pi}{2}} E_{tz} \sin \theta d\theta = \alpha^* \int_0^{\frac{\pi}{2}} E_{tt} \sin \theta d\theta \quad (15)$$

138 Compared to the more usual bar-type statistics (Bosquet, 1967; London, 1950), the star-type  
 139 statistics does not include the factor  $\cos \theta$ ; but there is no need to introduce this factor as  
 140  $E_{tz}$  already contains it, since it is the normal flux into the wall. Therefore is the star-type  
 141 statistics the obvious choice. Assuming as usual an isotropic distribution of wave directions,

142 the term  $|\partial_t \psi|^2$  is then constant for all wave directions in the first line of eq. (3) and in eq.  
 143 (4). The left hand side of eq. (15) simply reduces to the mean value of  $\sin \theta$ ; and the right  
 144 hand side also includes the mean value of  $\sin^3 \theta$ . Therefore, one obtains:

$$\alpha^* = \frac{2 \Re(\beta)}{1 + \frac{2}{3} + |\beta|^2} \quad (16)$$

145 which is the value of  $\alpha(\theta)$  taken for  $\theta = 55^\circ$ , an angle that has repeatedly appeared since  
 146 the 1950s as the incidence angle that represents the mean absorption (Bosquet, 1967; Guig-  
 147 nouard, 1991; Vogel, 1956). Indeed,  $\sin^2 55^\circ = 0.67 \approx \frac{2}{3}$ .

148 Simple calculation shows that the absorption coefficients given by eq. (5) and (16) take  
 149 values between 0 and a maximum reached when  $\Re(\beta) = \sqrt{1 + \sin^2 \theta + \Im^2(\beta)}$  and  $\Re(\beta) =$   
 150  $\sqrt{1 + \frac{2}{3} + \Im^2(\beta)}$  respectively. This maximum is respectively equal to  $\frac{1}{\sqrt{1 + \sin^2 \theta + \Im^2(\beta)}}$  and  
 151  $\frac{1}{\sqrt{1 + \frac{2}{3} + \Im^2(\beta)}}$ , that is, reaches its absolute maximum for  $\Im(\beta) = 0$  with value  $\frac{1}{\sqrt{1 + \sin^2 \theta}}$  and  
 152  $\frac{1}{\sqrt{1 + \frac{2}{3}}} \approx 0.775$  respectively. Both absolute maxima are therefore smaller than 1, except at  
 153 normal incidence ( $\theta = 0$ ) for the first one.

154 Last but not least, integrating eq. (16) on octave or third octave energy bands leads to  
 155 the usual definition of band related absorption coefficients.

## 156 B. Scattering coefficients

157 The scattering coefficients given by eq. (6) are independent of the incidence angle  $\theta$ ,  
 158 except through the reflection coefficient in the expression of  $\tan \phi_x$  and  $\tan \phi_y$  for curved  
 159 boundaries. However, the real part of the wall admittance contributes to the scattering

160 coefficients. In other words, absorption on the wall induces scattering. This is a novel  
161 result.

162 Also novel is the observation that scattering coefficients are not bounded: they can take  
163 any value between  $-\infty$  and  $+\infty$ , as is easily seen from eq. (6).

164 Traditionally, scattering is considered as the result of two mechanisms (Embrecchts, 2002):  
165 finite dimensions of reflecting surfaces; and roughness of the reflecting surfaces. There is no  
166 doubt from Sect. VC that the second term of eq. (6) corresponds to scattering by surface  
167 roughness, since it is driven by the finite distance from the surface to the image source.  
168 However, this term subsists for sources located at finite distances from a flat reflecting  
169 surface (see Sect. VB), as long as its admittance admits an imaginary part. But roughness  
170 is usually associated with locally varying imaginary parts of the admittance, as can be easily  
171 demonstrated with Schroeder diffusers, where the depths of the wells drive this imaginary  
172 part. In the present case, no locally varying admittance is necessary to induce scattering.

173 We therefore suspect that the mechanism described in Sect. V does not correspond  
174 to traditional scattering, but to a different effect. Indeed, in the work of Dujourdy *et al.*  
175 (Dujourdy *et al.*, 2017, 2019) and Meacham *et al.* (Meacham *et al.*, 2019), the scattering  
176 coefficient induces a decrease of steady-state energy with distance. Although scattering does  
177 induce a decrease of steady-state energy with distance by redirecting part of it toward the  
178 source, no redirection is here at stake, just a reduction of the acoustical intensity by friction  
179 on the walls. In other words, total energy and acoustical intensity can be considered as  
180 two coupled systems that each follow its own conservation equation with some exchange

181 between the two systems. As is well known from such coupled systems, the less damped  
 182 system drives the energy decay with time, which precisely is what is obtained in Fig. 1.

183 As a consequence, the scattering coefficient  $D$  is ill-named. A better designation would  
 184 be *intensity-friction* coefficient, or simply *friction* coefficient. For the time being, we stick  
 185 to usage and keep the designation "scattering coefficient".

186 Note that, as for the absorption coefficient, integrating eq. (6) on octave or third octave  
 187 energy bands leads to band related scattering - or friction - coefficients.

### 188 C. Admittance, impedance, and acoustic resistance

189 From the expression of the absorption and scattering coefficients (eqs. 5 and 6), it becomes  
 190 possible to express the (Replaced: ~~reduced~~ replaced with: *specific acoustic*) admittance in  
 191 the case of flat walls, which is the case of Fig. 1. From this expression, the (Replaced:  
 192 ~~reduced~~ replaced with: *specific acoustic*) impedance and the acoustic resistance of the wall  
 193 can also be computed.

194 Table I presents the value of the octave-band absorption and scattering coefficients of the  
 195 hallway of Fig. 1, derived from the figure itself by taking the centre of the vertical green  
 196 patches. (Added: There is no green patch in Fig. 1 for the 125 Hz octave band, but the two  
 197 stripes are tangent to each other at the point used in Table I.) Note that, according to eq.  
 198 (6), the real part of the (Replaced: ~~reduced~~ replaced with: *specific acoustic*) admittance is  
 199 equal to the scattering coefficient (plane walls). The imaginary part is obtained from eq. (5)  
 200 which gives valid values for all octave bands. Two possibilities exist here: either considering  
 201 a diffuse sound field, and use the average absorption coefficient of eq. (16); or considering

202 that the sound field remains parallel to the walls, and use eq. (5) with  $\sin \theta = 0$ . The latter  
 203 case gave more consistent results.

204 The (Replaced: ~~reduced~~ replaced with: **specific acoustic**) impedance is then computed  
 205 as the inverse of the (Replaced: ~~reduced~~ replaced with: **specific acoustic**) admittance, and  
 206 multiplying its real part by the characteristic impedance (Added: **of air**)  $\rho_0 c = 410 \text{ Ns/m}^3$   
 207 approximates the acoustic resistance of the wall - also in  $\text{Ns/m}^3$ . No attempt has been made  
 208 to compare the obtained values to typical walls, given that they are made of particle boards  
 209 and gyps boards, with a hard plastic floor. Indeed, quick evaluation of the resonances of  
 210 the walls by knocking on them revealed a complex structure with many resonances spread  
 211 over all the octave bands of interest, rendering inconsistent the parsing of the measured  
 212 impedance into resonances.

## 213 VII. CONCLUSION

214 Starting with the precedent papers of Dujourdy *et al.* (Dujourdy *et al.*, 2017, 2019)  
 215 that have shown that integrating the stress-energy tensor in a disproportionate enclosure  
 216 and taking into account the boundary conditions leads to "loss terms" that correspond to  
 217 absorption and scattering, we expressed the elements of the stress-energy tensor with the  
 218 help of the wall admittance relation between normal derivative and time derivative of the  
 219 velocity potential on the wall.

220 As expected, the absorption coefficients are proportional to the real part of the wall  
 221 admittance. However, they also depend on the angle of incidence of the wave, leading to an  
 222 expression of the absorption coefficient eq. (16) which has been signaled in the literature,

TABLE I. Admittance, impedance and acoustic resistance of the walls of the hallway of Fig. 1.

octave band	63Hz	125Hz	250Hz	500Hz	1kHz	2kHz	4kHz	8kHz
absorption coeff.	0.025	0.115	0.1	0.095	0.09	0.12	0.14	0.13
scattering coeff.	0.6	0.125	0.335	0.355	0.22	0.135	0.145	0.23
$\Re(\beta)$	0.6	0.125	0.335	0.355	0.22	0.135	0.145	0.23
$\Im(\beta)$	6.83	1.08	2.036	2.52	1.96	1.11	1.02	1.58
$\Re(\zeta)$	0.013	0.106	0.059	0.055	0.057	0.108	0.135	0.091
$\Im(\zeta)$	-0.15	-0.92	-0.41	-0.39	-0.50	-0.89	-0.96	-0.62
resistance (Ns/m <sup>3</sup> )	5.23	43.7	24.1	22.5	23.2	44.3	55.5	37.1

223 for example by Bosquet (Bosquet, 1967), yet is not very usual; and to scattering coefficients  
 224 additively involving the real and imaginary parts of the wall admittance, as seen in eq.  
 225 (6), proving the absorptive nature of the scattering coefficient in the stress-energy tensor  
 226 formalism.

227 In the case of plane boundaries, calculations can be extended to evaluate the mean  
 228 impedance of the walls. An example is given for a long hallway.

229 **ACKNOWLEDGMENTS**230 **AUTHOR DECLARATIONS**231 **Conflict of Interest**

232 The authors have no conflicts to disclose.

233 **Data Availability**

234 Data sharing not applicable no new data generated.

235

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## List of Changes

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Added: The stress-energy tensor formalism was initially introduced by Morse and collaborators (Morse and Feshbach, 1956; Morse and Ingard, 1968) in order to describe energy conservation in linear acoustics. Many years later, Stanzial and collaborators applied the conservation of the stress-energy tensor to room acoustics (Stanzial *et al.*, 2002), focusing on the expression of the radiation pressure. In a subsequent paper (Stanzial and Schiffrer, 2010), they introduced the energy velocity as ratio of the sound intensity to the total acoustic energy, and derived from the conservation of the stress-energy tensor an expression for the local initial reverberation time at the onset of energy decay. Some related ideas, linking wave impedance to the complex sound intensity, are also found

in (Mann *et al.*, 1987; Stanzial and Graffigna, 2017). Mann *et al.* focused on energy transfer and analysed in terms of energy flux and power what happens in the vicinity of intensity vortex that are created by specific configurations of sources. Stanzial and Graffigna went one step further in the definition and measurement of complex intensity. However, both papers are outside the scope of the present analysis that focuses on the active stress-energy tensor - see (Polack, 2023) for an extension to the *complex* stress-energy tensor and its conservation, together with preliminary measurement results., on page 2, line 12.

Replaced: ~~The stress-energy tensor formalism~~ replaced with: A somewhat different perspective, on page 2, line 27.

Added: in order, on page 2, line 28.

Added: steady-state, on page 4, line 50.

Replaced: ~~reduced~~ replaced with: specific acoustic, on page 5, line 67.

Added: (Morse and Ingard, 1968, p. 580), on page 5, line 67.

Added: that is, the wall impedance normalized by the characteristic impedance  $\rho_0 c$  of air, on page 5, line 67.

Added: is, on page 6, line 67.

Replaced: ~~reduced~~ replaced with: specific acoustic, on page 6, line 67.

Added: Note that  $E_{xz}$  and  $E_{yz}$  correspond to the parallel components of the radiation pressure exerted on the "ceiling" and the "floor" (Stanzial *et al.*, 2002)., on page 7, line 69.

Added: (Morse and Ingard, 1968, p. 580), on page 15, line 130.

Replaced: ~~reduced~~ replaced with: **specific acoustic**, on page 18, line 189.

Replaced: ~~reduced~~ replaced with: **specific acoustic**, on page 18, line 190.

Added: **There is no green patch in Fig. 1 for the 125 Hz octave band, but the two stripes are tangent to each other at the point used in Table I.**, on page 18, line 195.

Replaced: ~~reduced~~ replaced with: **specific acoustic**, on page 18, line 197.

Replaced: ~~reduced~~ replaced with: **specific acoustic**, on page 19, line 203.

Replaced: ~~reduced~~ replaced with: **specific acoustic**, on page 19, line 204.

Added: **of air**, on page 19, line 205.