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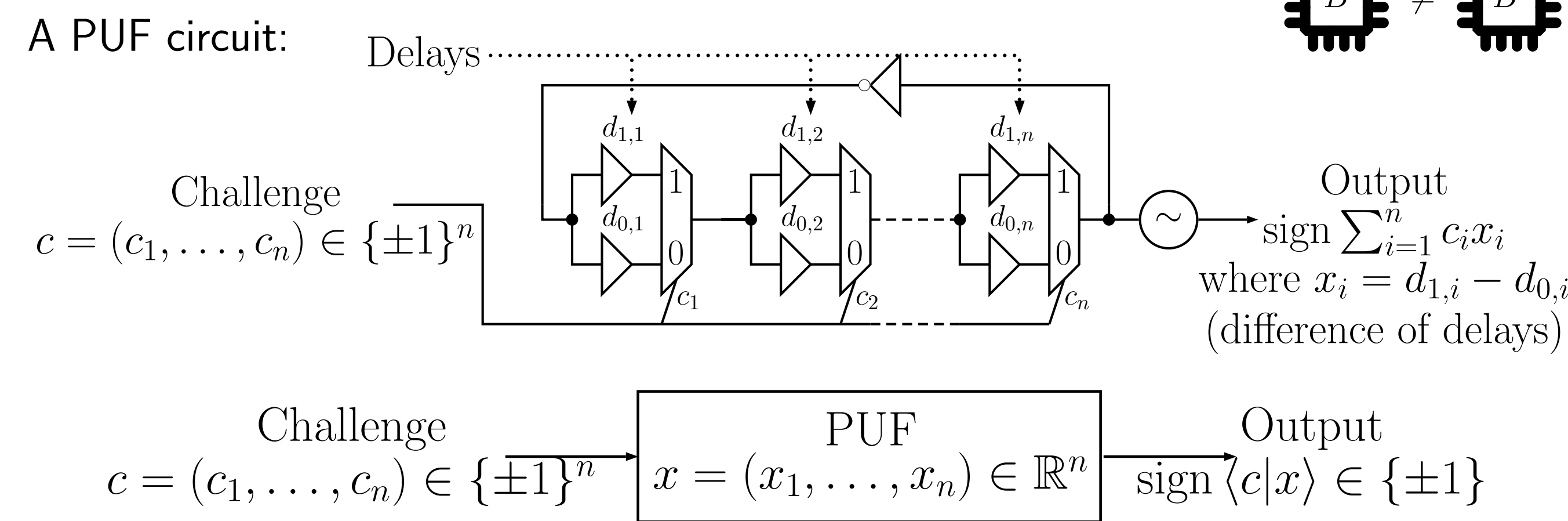


# Rényi Entropy Estimation for Secure Silicon Fingerprints

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## Physically Unclonable Function (PUF)



The PUF parameter  $x$  is **uncontrollable**: it depends on the manufacturing process and varies from one circuit to another ("static randomness").

$x$ : outcome of an **i.i.d random vector**  $X = (X_1, \dots, X_n)$  where  $X_i \sim -X_i$ .

The PUF behavior depends on a **challenge code**

$$\mathcal{C} = \{c^1, \dots, c^M\}$$

that defines the PUF output (**silicon fingerprint**)

$$B = (B_1, \dots, B_M) \text{ where } B_j = \text{sign} \langle c^j | X \rangle.$$

## Motivation

Given a PUF design:

- How much **randomness** can be extracted from it? More precisely:
- How **unique** is the generated identifier (collision resistance)?
- Can a PUF be used to generate a (say) 128-bit **cryptographic key**?
- How difficult is it to **predict** the PUF response (using machine learning, e.g., SVM)?

## Rényi Entropy as a PUF Security Metric

### $\alpha$ -Entropy

$$H_\alpha(B) = \frac{1}{1-\alpha} \sum_{b \in \{\pm 1\}^M} P_B(b)^\alpha$$

Definition	Name	Significance
$H_0(B) = \log \#\text{Supp}(B)$	max-entropy	Number of PUFs
$H_1(B) = H(B)$	Shannon entropy	Probabilistic uncertainty
$H_2(B) = -\log \mathbb{P}(B = B'),$ $B \perp\!\!\!\perp B'$	Collision entropy	Resistance to random collisions
$H_\infty(B) = -\log \max_b P_B(b)$	min-entropy	Brute-force resistance

- $H_\alpha(B_1, \dots, B_M)$  **increases** with  $M$  (when adding new challenges to  $\mathcal{C}$ ).
- $\max_{\mathcal{C}} H_\alpha(B) = H_\alpha(n)$  is attained for  $M = 2^n$  (all possible challenges).

## BTF Theory and Chow Parameters

A PUF with  $M = 2^n$  is identified to a

### Boolean Threshold Function (BTF)

$$f_x : c \in \{\pm 1\}^n \mapsto \text{sign} \langle c | x \rangle \text{ for a fixed } x \in \mathbb{R}^n.$$

### Chow parameters of a BTF

$$\hat{f}_x = \sum_{f_x(c)=1} c \text{ (componentwise).}$$

They **uniquely identify** a boolean threshold function:  $\hat{f}_x = \hat{f}_y \implies f_x = f_y$ .

### Result on the max-entropy

There are at most  $2^{n^2}$  distinct Chow parameters. Hence  $H_0(n) < n^2$  ( $\forall n \geq 2$ ).

## Invariance Under the Signed Permutation Group

Changing the **order** or **signs** of the  $X_i$ 's does not change  $P_B$ .

### Signed Permutation Group:

$G_n = S_n \times \{\pm 1\}^n$  acts on  $\mathbb{R}^n$ :

$$(g \cdot x)_i = s_i x_{\sigma(i)} \text{ where } g = (\sigma, s) \in S_n \times \{\pm 1\}^n.$$

### Results on equal probabilities

- The action of  $G_n$  preserves probabilities:  $\mathbb{P}(f_X = f_x) = \mathbb{P}(f_X = f_{g \cdot x})$
- Hence all PUFs in  $\text{Orb}(f_x) = \{f_{g \cdot x} \mid g \in G_n\}$  have the same probability.

### Results on the orbit size

- The action of  $G_n$  preserves Chow parameters:  $\hat{f}_{g \cdot x} = g \cdot \hat{f}_x$
- Hence, using the orbit-stabilizer theorem on  $\text{Orb}(f_x)$ ,

$$\#\text{Orb}(f_x) = \frac{2^n n!}{2^{m_{\hat{f}_x}(0)} \prod_{k \in \mathbb{N}} m_{\hat{f}_x}(k)!} \text{ where } m_{\hat{f}_x}(k) = \#\{i : (\hat{f}_x)_i = \pm k\}.$$

## Estimating the PUF Distribution

for  $i \leftarrow 1$  to  $N$  do

Generate  $n$  realizations  $x = (x_1, \dots, x_n)$ ;  
Take  $x \leftarrow |x|$  and sort  $x$  to get the orbit leader;  
Compute Chow parameters  $\hat{f}_x$ ;  
Set **count**  $[\hat{f}_x] \leftarrow$  **count**  $[\hat{f}_x] + 1$ ;

end

for  $c \in$  **count** do

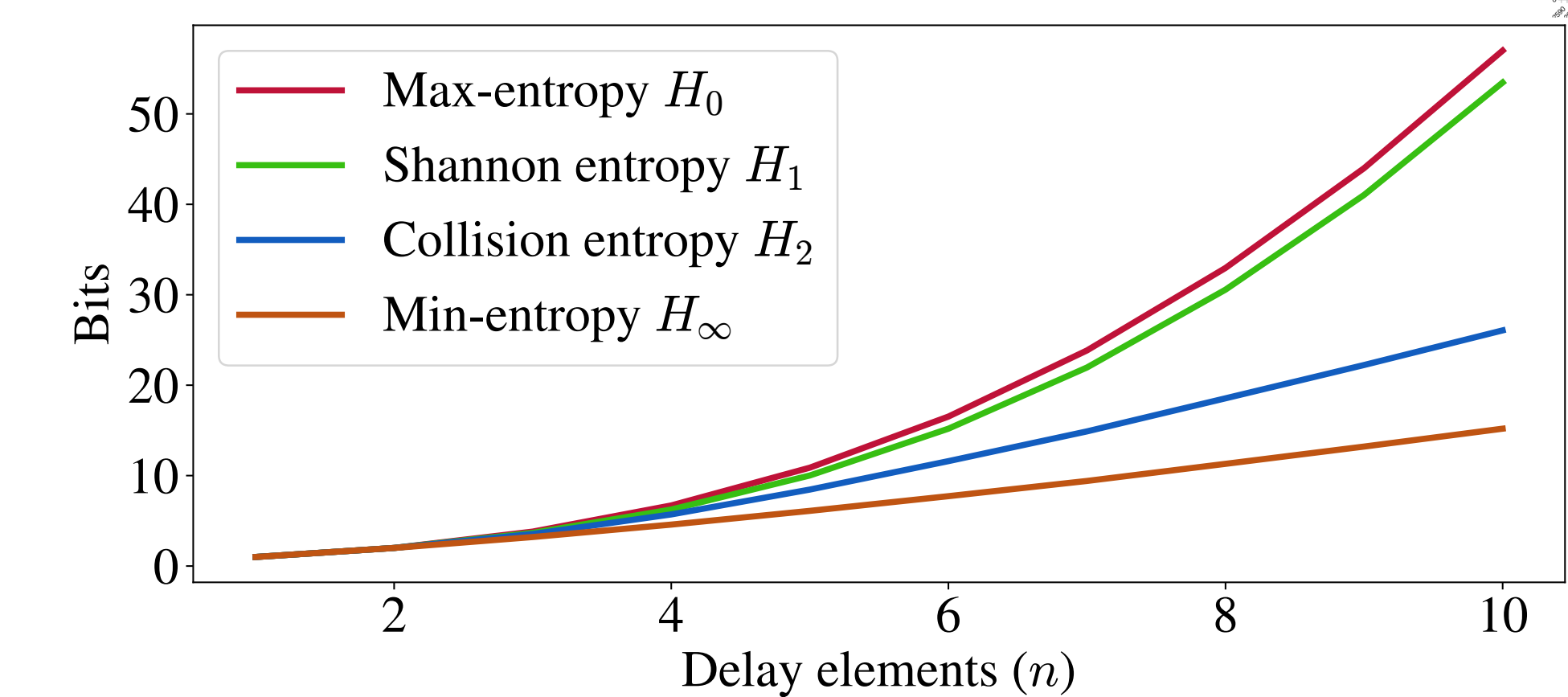
$$\text{orbit\_size}[c] \leftarrow \frac{2^n n!}{2^{m_c(0)} \prod_{k \in \mathbb{N}} m_c(k)!}, \quad \text{proba}[c] \leftarrow \frac{\text{count}[c]}{N * \text{orbit\_size}[c]};$$

end

### Simulation interpretation

For any orbit leader  $f_x$ , there are **orbit\_size** $[\hat{f}_x]$  probabilities equal to **proba** $[\hat{f}_x]$ .

## Simulation Results for Gaussian $X$



### Results with 95 % confidence level

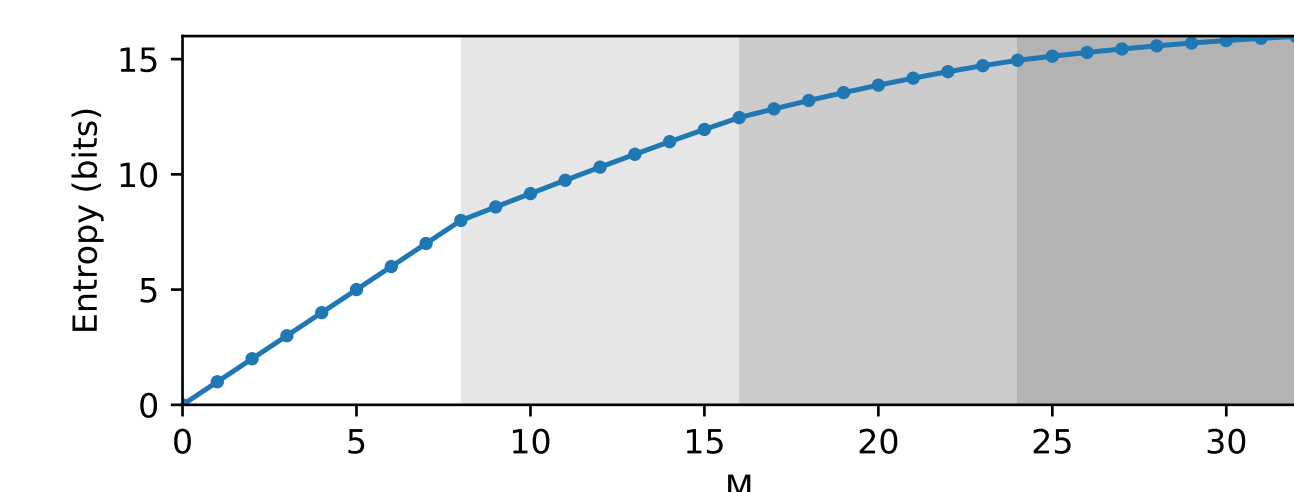
$n$	Sample size	$H_1(n)$ (bits)	$H_2(n)$ (bits)	$H_\infty(n)$ (bits)
3	N/A	3.6655...	3.5462...	3.2086...
4	N/A	6.2516...	5.7105...	4.5850...
5	$10^{10}$	$10.014 \pm 1 \cdot 10^{-3}$	$8.4559 \pm 8 \cdot 10^{-4}$	$6.1007 \pm 1 \cdot 10^{-4}$
6	$10^{10}$	$15.191 \pm 1 \cdot 10^{-3}$	$11.600 \pm 2 \cdot 10^{-3}$	$7.7353 \pm 1 \cdot 10^{-4}$
7	$10^{10}$	$21.987 \pm 1 \cdot 10^{-3}$	$14.890 \pm 8 \cdot 10^{-3}$	$9.4733 \pm 2 \cdot 10^{-4}$
8	$2 \cdot 10^{10}$	$30.5636 \pm 8 \cdot 10^{-4}$	$18.55 \pm 3 \cdot 10^{-2}$	$11.3022 \pm 2 \cdot 10^{-4}$
9	$2 \cdot 10^{10}$	$41.0376 \pm 8 \cdot 10^{-4}$	$22.2 \pm 2 \cdot 10^{-1}$	$13.2127 \pm 4 \cdot 10^{-4}$
10	$3 \cdot 10^{12}$	$53.4738 \pm 1 \cdot 10^{-4}$	$26.1 \pm 1 \cdot 10^{-1}$	$15.1900 \pm 1 \cdot 10^{-4}$

## Conclusion and Perspectives

Cryptographic applications require a source of high entropy.

We show how to calibrate PUFs to achieve a given security level:

- Our simulation results suggest that  $H_1(n) \approx n^2$  (much greater than  $n$ )!
- Exact computation of entropies for higher values of  $n$  becomes intractable.
- But in practice, only a subset of all possible challenges is chosen:  
How should we select them to maximize entropy?



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