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# A new parametrization for the Rician distribution

Jean-Marie-Nicolas and Florence Tupin, *Senior Member, IEEE*

**Abstract**—The Rician distribution is widely used in SAR imagery to model the backscattering of a strong target inside a resolution cell. Nevertheless the computation of the parameters of the Rice distribution remains a difficult task. In this paper, a new parametrization to model the Rice distribution is introduced. Thanks to the introduction of a new variable defined by the ratio of the target contribution to the speckle, the relationship between the coefficient of variation and this new parameter can be derived. An efficient numerical method is proposed to evaluate it from the coefficient of variation and a discussion on the variance of this estimator is led. A comparison with other methods of estimation showed that the proposed approach is a good compromise between the variance of the estimate and the computation time. At last, a link between permanent scatterers and Rice distributed targets is proposed through this new parametrization.

**Index Terms**—Statistical modeling, the Rician distribution, SAR data.

## I. INTRODUCTION

SAR imagery has become very popular in the past years thanks to its all time and all weather capacities. The recent launch of Sentinel-1 and ESA's data policy makes very long time series now freely available. These temporal series open new ways to understand SAR data distributions. Indeed, the local stationarity hypothesis, essential to analyse speckle and texture distributions with a single SAR image, is no longer necessary when dealing with time series. In some conditions (no change case), the temporal samples can be considered as independent and identically distributed samples and can be used to model the statistical behaviour of the different areas.

This statistical point is of particular interest when dealing with point-like targets, where no local estimation can be applied. Indeed, neighboring pixels follow a different probability density function (pdf) and can not be used to compute any statistics. In this situation, with a single image, only one sample is available to evaluate pdf parameters leading to degenerate estimations. When having long time series, if the samples are decorrelated and temporally stable (meaning no change in the imaged physical area), the temporal samples can be used as samples to evaluate the pdf parameters. During many decades, the number of samples was relatively small to allow parameter estimation with a low variance. But recently, a huge number of images can be used (more than 100 data). This situation allows new investigations on the pdf followed in SAR imagery, specially for point-like targets or areas with a very small extension.

In this paper we are interested in the Rician distribution widely used in SAR imagery to model strong scatterers

[1][2][3][4][5]. This pdf is thus of particular interest when dealing with urban areas with many strong echoes due to man-made structures with dihedral or trihedral configurations.

The paper is organized as follows. In section II, we recall the Rician distribution and some common methods for parameter estimation. In section III, we introduce a new formulation of the Rician pdf through the definition of a parameter relying on the ratio between the strong scatterer contribution and the underlying speckle reflectivity. We then present a new method for parameter estimation based on this parametrization and compare different parameter estimates. We then discuss the link between permanent scatterers and Rician pdf.

## II. THE RICIAN DISTRIBUTION

### A. The Rician distribution and its moments

In Radar imaging, acoustical imaging and laser analysis, the speckle effect is a rather well known phenomenon based on the coherent nature of the illumination which affects the quality of the images. "Fully developed speckle" [1] assumes a large distribution of quasi point-like identical backscattering targets so that the probability density function of the data for a physically constant area can be modelled by a Rayleigh distribution (in the case of amplitude data) or negative exponential distribution (in the case of intensity data). These first order statistical properties can be easily deduced from the study of random phasor sums [1].

In the radar framework, Rice's works were devoted to the distribution of noise on a sine wave [6], yielding an interesting relationship for the probability density function for a deterministic target associated with speckle noise (illustrated in figure 1). If  $\mu_C$  is the amplitude of the deterministic signal and  $\mu$  is the parameter of the Rayleigh distribution associated with the speckle noise, the probability associated to a given value  $x$  can be written as [6] (with  $\mu$  and  $\mu_C \in \mathbb{R}^+$ ):

$$\mathcal{RC}[\mu, \mu_C](x) = \frac{2x}{\mu^2} e^{-\frac{x^2 + \mu_C^2}{\mu^2}} I_0\left(\frac{2\mu_C x}{\mu^2}\right), \quad (1)$$

where  $I_0$  is the modified Bessel function of the first kind. This relation is known as the "Rician" density function.

As we have:

$$I_0(y) \simeq 1 + \left(\frac{y}{2}\right)^2, \quad (2)$$

we can derive the following property :

$$\lim_{\mu_C \rightarrow 0} \mathcal{RC}[\mu, \mu_C](x) = \frac{2x}{\mu^2} e^{-\frac{x^2}{\mu^2}}, \quad (3)$$

i.e. the Rician distribution tends to the Rayleigh distribution.

As  $\frac{\mu_C}{\mu}$  grows, the Rician distribution takes on a more symmetrical form and is asymptotically Gaussian [1] (Fig.2).

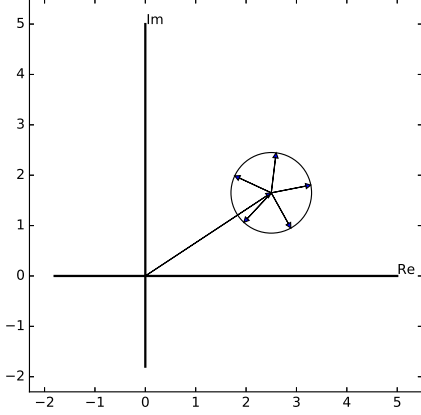


Fig. 1. Example of the coherent sum of a strong deterministic backscatterer with amplitude  $\mu_C$  and speckle of small amplitude  $\mu$  compared to  $\mu_C$ , leading to observations lying in the represented circle.

The moments of the Rician distribution are easily seen to be [1]:

$$\begin{cases} m_1 = \mu \frac{\sqrt{\pi}}{2} e^{-\frac{\mu_C^2}{\mu^2}} {}_1F_1\left(\frac{3}{2}; 1; \frac{\mu_C^2}{\mu^2}\right), \\ m_2 = \mu^2 e^{-\frac{\mu_C^2}{\mu^2}} {}_1F_1\left(2; 1; \frac{\mu_C^2}{\mu^2}\right), \\ m_r = \mu^r \Gamma\left(1 + \frac{r}{2}\right) e^{-\frac{\mu_C^2}{\mu^2}} {}_1F_1\left(1 + \frac{r}{2}; 1; \frac{\mu_C^2}{\mu^2}\right), \end{cases} \quad (4)$$

where  ${}_1F_1$  is the confluent hypergeometric function, also called Kummer function. By this way, it is possible to derive the coefficient of variation  $\gamma$ , widely used for SAR imagery due to the multiplicative noise modeling [7] [8] [9]:

$$\gamma = \sqrt{\frac{m_2}{m_1^2} - 1} = \sqrt{\frac{4 e^{\frac{\mu_C^2}{\mu^2}} {}_1F_1\left(2; 1; \frac{\mu_C^2}{\mu^2}\right)}{\pi \left({}_1F_1\left(\frac{3}{2}; 1; \frac{\mu_C^2}{\mu^2}\right)\right)^2} - 1}. \quad (5)$$

Unfortunately, the coefficient of variation depends on both parameters  $\mu$  and  $\mu_C$ .

### B. Estimation of the Rician distribution parameters

In order to estimate the parameters of the Rician distribution, several methods can be investigated :

- The method of moments using the two first moments (see equation 4). In this case, no explicit expressions involving these moments  $m_1$  and  $m_2$  can provide  $\mu$  and  $\mu_C$ . Only a numerical scheme allows this inversion.
- The method of moments using the second moment and the fourth one. Knowing the following properties for the Kummer function  ${}_1F_1$  [10] :

$$\begin{aligned} {}_1F_1(2; 1; x) &= (1+x) e^x, \\ {}_1F_1(3; 1; x) &= \frac{1}{2} (2 + 4x + x^2) e^x, \end{aligned}$$

the equation (4) provides simple expressions of the second moment and the fourth one :

$$\begin{cases} m_2 = \mu^2 + \mu_C^2, \\ m_4 = 2\mu^4 + 4\mu_C^2\mu^2 + \mu_C^4. \end{cases} \quad (6)$$

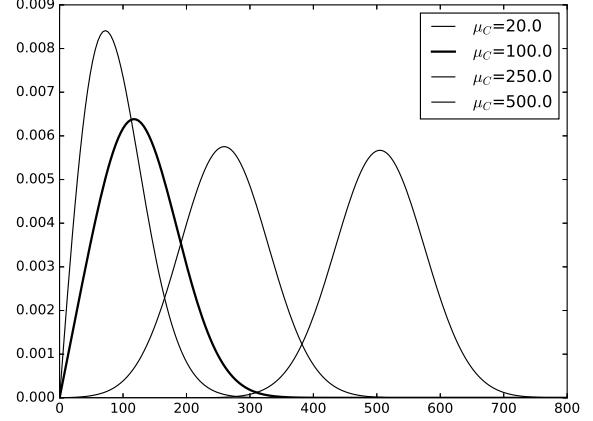


Fig. 2. Rician distribution for a given speckle ( $\mu = 100$ ) and for various values of  $\mu_C$ . For  $\mu_C \rightarrow 0$ , the Rician distribution tends to a Rayleigh distribution.

Knowing the second and fourth moments :  $m_2$  and  $m_4$ , we obtain the estimates of the two parameters  $\mu_C$  and  $\mu$  by the following explicit equations:

$$\mu_C = (2m_2 - m_4)^{\frac{1}{4}}, \quad (7)$$

$$\mu = \left(m_2 - (2m_2 - m_4)^{\frac{1}{2}}\right)^{\frac{1}{2}}. \quad (8)$$

- The Maximum Likelihood estimation method, which can only be used with an iterative scheme (see [11]).

All these methods can provide correct estimation of  $\mu$  and  $\mu_C$ . Meanwhile these two parameters are strongly linked in this parametrization so that all classical statistical values (moments, cumulants, the coefficient of variation, ...) depend on both the two parameters  $\mu$  and  $\mu_C$ .

## III. A NEW PARAMETRIZATION FOR THE RICIAN DISTRIBUTION

### A. Theory

If the Rician distribution is traditionally described with  $\mu$  (the speckle noise) and  $\mu_C$  (the target), the key idea of this new parametrization is to introduce the ratio between these two parameters. By this way, we consider two parameters :  $\mu$  (the speckle noise) and a new parameter  $\lambda$  so that

$$\lambda = \frac{\mu_C}{\mu} \Leftrightarrow \mu_C = \lambda \mu. \quad (9)$$

$\lambda$  can be viewed as a kind of tuning parameter concerning the deterministic target: the higher the parameter, the stronger the effects of the target. It thus has an intuitive physical meaning, measuring the relative strength of the target compared to the speckle contribution. Taking into account the link with random walks with drift to describe the speckle and the deterministic target, the parameter  $\lambda$  will be called *relative drift* in the following.

The Rician distribution from equation (1) can be rewritten as:

$$\mathcal{RC}_2[\mu, \lambda](x) = \frac{2x}{\mu^2} e^{-\left(\frac{x^2}{\mu^2} + \lambda^2\right)} I_0\left(2\lambda \frac{x}{\mu}\right). \quad (10)$$

Knowing the relation given by equation (2), we can derive the following property:

$$\lim_{\lambda \rightarrow 0} \mathcal{RC}_2[\mu, \lambda](x) = \frac{2x}{\mu^2} e^{-\frac{x^2}{\mu^2}}, \quad (11)$$

i.e. the Rician distribution tends to the Rayleigh distribution if the target vanishes.

In this new framework, the moments can be written as:

$$\begin{cases} m_1 = \mu e^{-\lambda^2} \Gamma(1 + \frac{1}{2}) {}_1F_1(1 + \frac{1}{2}; 1; \lambda^2), \\ m_2 = \mu^2 e^{-\lambda^2} {}_1F_1(2; 1; \lambda^2) = \mu^2 (1 + \lambda^2), \\ m_r = \mu^r e^{-\lambda^2} \Gamma(1 + \frac{r}{2}) {}_1F_1(1 + \frac{r}{2}; 1; \lambda^2). \end{cases} \quad (12)$$

Moreover, the coefficient of variation  $\gamma$  depends only on  $\lambda$ :

$$\gamma = \sqrt{\frac{e^{\lambda^2} {}_1F_1(2; 1; \lambda^2)}{(\Gamma(1 + \frac{1}{2}) {}_1F_1(1 + \frac{1}{2}; 1; \lambda^2))^2} - 1}, \quad (13)$$

and, with the help of another property of Kummer functions [10]:

$${}_1F_1\left(\frac{3}{2}; 1; x\right) = e^{\frac{x}{2}} \left( (1+x) I_0\left(\frac{x}{2}\right) + x I_1\left(\frac{x}{2}\right) \right),$$

(where  $I_0$  and  $I_1$  are modified Bessel functions of the first kind), the following expression can be easily derived:

$$\gamma = \sqrt{\frac{4 e^{\lambda^2} (1 + \lambda^2)}{\pi \left( (1 + \lambda^2) I_0\left(\frac{\lambda^2}{2}\right) + \lambda^2 I_1\left(\frac{\lambda^2}{2}\right) \right)^2} - 1}, \quad (14)$$

so that, for  $\lambda \in [0, \infty[$ , we have  $\gamma \in [0, \sqrt{\frac{4-\pi}{\pi}}]$ .

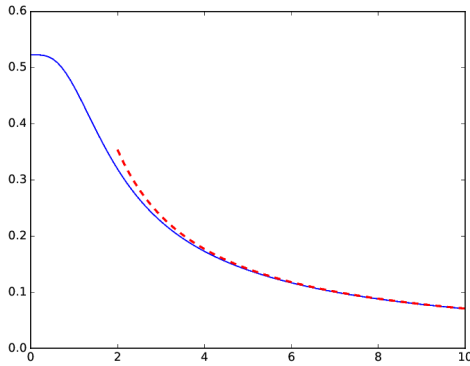


Fig. 3. Variation coefficient  $\gamma$  of the Rician distribution  $\mathcal{RC}_2[\mu, \lambda]$  for various values of  $\lambda$  as given by the equation (14). The asymptotic behavior (red curve) when  $\lambda \rightarrow \infty$  is  $\frac{1}{\sqrt{2\lambda}}$ .

### B. A new method to estimate the Rician distribution parameters ( $M_\gamma$ method)

The main point of this new approach is the fact that the coefficient of variation depends only on one unique parameter:  $\lambda$ . Meanwhile the mathematical expression (14) is an implicit expression and no analytical relation can provide  $\lambda$  as a function of  $\gamma$ .

Nevertheless, if we plot  $\gamma$  as a function of the relative drift  $\lambda$  (figure 3), it seems to be possible to invert numerically this relation. By using a RPM (Rational Polynomial Model) [12], we propose the following approximation  $\tilde{\lambda}$  given by equation (15).

By this approach, it is possible to deduce the Rician distribution parameters knowing the two first moments :

- we obtain  $\tilde{\lambda}$ , an approximation of the relative drift  $\lambda$  with the help of the equation (15)
- then, an approximation of  $\mu$ ,  $\tilde{\mu}$ , can be derived from equation (12) :

$$\tilde{\mu} = \sqrt{\frac{m_2}{1 + \tilde{\lambda}^2}}. \quad (16)$$

Let us remark that, knowing some properties of modified Bessel functions, it is easy to derive the following asymptotic properties from the equation (14) :

$$\begin{cases} \gamma|_{\lambda \rightarrow \infty} \simeq \frac{1}{\sqrt{2\lambda}}, \\ \gamma|_{\lambda \rightarrow 0+} \simeq 0.52272 - 0.15224 \lambda^4. \end{cases} \quad (17)$$

Moreover, as the variance of the coefficient of variation is given by the equation (derived from the definition of the coefficient of variation and Kendall&Stuart method [13]) :

$$\text{var}(\gamma) = \frac{1}{N} \frac{1}{4} \frac{4m_2^3 - m_2^2 m_1^2 + m_1^2 m_4 - 4m_1 m_2 m_3}{m_1^4 (m_2 - m_1^2)}. \quad (18)$$

with  $N$  the number of samples used for the estimation, this variance depends only on the  $\lambda$  parameter in the case of a Rician distribution modelled with  $\mu$  and  $\lambda$ .

Figure 4 shows examples of Rice histograms and the fitting pdf obtained with  $\lambda$  estimation. As it can be observed, the Rice distribution looks like a Rayleigh distribution when the relative drift  $\lambda$  is small and tends towards a Gaussian distribution when  $\lambda$  increases.

### C. Comparison of parameter estimation methods

To demonstrate the computational interest of using equation (15) for  $\lambda$  estimation, we compare different estimation methods:

- method of moments (MM)
- maximum likelihood (ML) method
- estimation based on the coefficient of variation ( $M_\gamma$ )

We assume that we have  $N$  samples  $x_i$ , yielding sample moments  $\hat{m}_r : \hat{m}_r = \langle \{x^r\} \rangle = \frac{1}{N} \sum_{i=1}^N x_i^r$ . For these three methods we aim to obtain  $\hat{\lambda}$  and  $\hat{\mu}$ , the estimates of  $\lambda$  and  $\mu$ .

For the method of moments, it can be shown (using the properties of Kummer functions [10] [14]) that we have the following equations :

$$\begin{cases} m_2 = \mu^2 (1 + \lambda^2), \\ m_4 = \mu^4 (2 + 4\lambda^2 + \lambda^4), \end{cases} \quad (19)$$

leading to the moment estimates:

$$\begin{cases} \hat{\lambda} = \frac{(2 \hat{m}_2^2 - \hat{m}_4)^{\frac{1}{4}}}{\left( \hat{m}_2 - (2 \hat{m}_2^2 - \hat{m}_4)^{\frac{1}{2}} \right)^{\frac{1}{2}}}, \\ \hat{\mu} = \left( \hat{m}_2 - (2 \hat{m}_2^2 - \hat{m}_4)^{\frac{1}{2}} \right)^{\frac{1}{2}}. \end{cases} \quad (20)$$

$$\tilde{\lambda} \simeq \frac{1 - 2.864928\gamma - 3.193363\gamma^2 + 15.715797\gamma^3 - 11.713746\gamma^4}{-0.017883 + 1.815109\gamma - 7.318177\gamma^2 + 8.717601\gamma^3 - 2.362803\gamma^4} \quad (15)$$

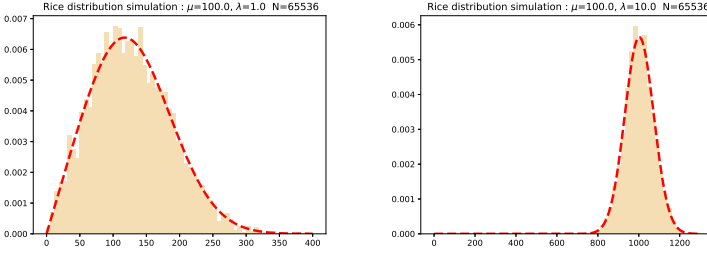


Fig. 4. Two simulations of Rician pdf for two  $\lambda$  parameters ( $\lambda = 1$  and 10 from left to right), with  $\mu = 100$  and  $N = 65536$  samples). For small  $\lambda$  values, the behaviour is close to a Rayleigh-Nakagami law (and tends towards it for  $\lambda \rightarrow 0$ ). For high  $\lambda$  values, the pdf is close to a Gaussian pdf.

For the maximum likelihood estimates, it can be shown that the following equations hold :

$$\begin{cases} \hat{\lambda} = \frac{1}{N} \sum_{i=1}^N \frac{x_i}{\hat{\mu}} \frac{I_1\left(\frac{2\hat{\lambda}x_i}{\hat{\mu}}\right)}{I_0\left(\frac{2\hat{\lambda}x_i}{\hat{\mu}}\right)}, \\ \hat{\mu}_2 = \hat{\mu}^2 (1 + \hat{\lambda}^2). \end{cases} \quad (21)$$

An iterative scheme can be defined from these two equations to derive the ML estimates.

In the case of high  $\lambda$  values, it is possible to consider an approximation of the Rice distribution by a Normal distribution. In this case, we have the following estimates:

$$\hat{\mu}_C = \hat{m}_1, \quad (22)$$

giving :

$$\begin{cases} \hat{\lambda} = \frac{\hat{m}_1}{\sqrt{2(\hat{m}_2 - \hat{m}_1^2)}}, \\ \hat{\mu} = \sqrt{2(\hat{m}_2 - \hat{m}_1^2)}. \end{cases} \quad (23)$$

This method based on the Gaussian approximation of the pdf will be denoted by MN. We give in the following tables (table I for lower values of  $\lambda$  and table II for highest values of  $\lambda$ ) a comparison of these estimation methods. The number of temporal samples to compute an estimation is indicated by  $N$ , and  $R = 128$  repeats are done to compute statistics on the estimators. Indeed, the  $R$  estimated values of  $\hat{\lambda}$  and  $\hat{\mu}$  are used to compute the averaged values ( $\bar{\lambda}$  and  $\bar{\mu}$ ), the Mean Square Errors (referred to as  $\sigma_\lambda^2$  and  $\sigma_\mu^2$  and measuring precision) and the Mean Absolute Errors (referred to as  $E_\lambda$  and  $E_\mu$  and measuring accuracy).

By varying the number of temporal samples  $N$  (from  $N = 100$  to  $N = 10^4$ ), it is also possible to verify that  $\sigma_\lambda \propto \frac{1}{\sqrt{N}}$  and  $\sigma_\mu \propto \frac{1}{\sqrt{N}}$ . For example, we obtain for the  $M_\gamma$  method for  $\lambda = 4$ , and  $\mu = 100$  :

$$\sigma_\lambda \sim \frac{2.81}{\sqrt{N}}, \text{ and } \sigma_\mu \sim \frac{66.}{\sqrt{N}}. \quad (24)$$

The following conclusions can be given. First, it can be computed from the tables that there are no significant bias in

	Speckle $\mu = 100.0$		N=100	
	$\lambda=1.0$		$\lambda=3.0$	
ML	$\bar{\lambda} = 1.038$ $\sigma_\lambda = 0.243$ $E_\lambda = 0.204$	$\bar{\mu} = 98.491$ $\sigma_\mu = 12.056$ $E_\mu = 9.564$	$\bar{\lambda} = 3.049$ $\sigma_\lambda = 0.249$ $E_\lambda = 0.206$	$\bar{\mu} = 98.988$ $\sigma_\mu = 7.068$ $E_\mu = 6.012$
MM	$\bar{\lambda} = 1.050$ $\sigma_\lambda = 0.302$ $E_\lambda = 0.237$	$\bar{\mu} = 97.711$ $\sigma_\mu = 13.674$ $E_\mu = 10.745$	$\bar{\lambda} = 3.056$ $\sigma_\lambda = 0.264$ $E_\lambda = 0.221$	$\bar{\mu} = 98.853$ $\sigma_\mu = 7.356$ $E_\mu = 6.360$
$M_\gamma$	$\bar{\lambda} = 1.019$ $\sigma_\lambda = 0.271$ $E_\lambda = 0.210$	$\bar{\mu} = 99.088$ $\sigma_\mu = 12.167$ $E_\mu = 9.440$	$\bar{\lambda} = 3.065$ $\sigma_\lambda = 0.248$ $E_\lambda = 0.208$	$\bar{\mu} = 98.505$ $\sigma_\mu = 6.993$ $E_\mu = 6.006$

TABLE I  
COMPARISON OF THE ESTIMATES OF RICIAN PARAMETERS FOR SMALL  $\lambda$  VALUES WITH THE METHODS: ML, MM,  $M_\gamma$  (128 REPEATS). THE PARAMETERS ARE  $\mu = 100$ . AND  $\lambda = 1$  AND 3. THE NUMBER OF SAMPLES IS 100. FOR THE ML METHOD, TO REACH CONVERGENCE 44 ITERATIONS ARE NECESSARY FOR  $\lambda = 1$ , AND 14 FOR  $\lambda = 3$ .

the estimations (with 95% confidence). Concerning precision, the ML estimates usually provide the minimum variance as expected. Nevertheless, for  $\lambda$  lower than 6., the proposed method based on the coefficient of variation gives quite close results for the precision, whereas the MM method gives the worst results due to the computation of higher order moments. For high  $\lambda$  values, it is recommended to use the Normal approximation of the Rice pdf and equation (23) for parameter estimation.

Concerning the computation time, the computational burden of the iterative process necessary for ML estimates is high. Indeed, it generally requires more than one hundred iterations and involves two Bessel functions computations at each iteration and for each sample; at the same time, the  $M_\gamma$  methods

	Speckle $\mu = 100.0$		N=100	
	Cas $\lambda=4.0$		Cas $\lambda=7.5$	
ML	$\bar{\lambda} = 4.052$ $\sigma_\lambda = 0.305$ $E_\lambda = 0.253$	$\bar{\mu} = 99.335$ $\sigma_\mu = 6.834$ $E_\mu = 5.864$	$\bar{\lambda} = 7.512$ $\sigma_\lambda = 0.517$ $E_\lambda = 0.463$	$\bar{\mu} = 100.271$ $\sigma_\mu = 6.737$ $E_\mu = 5.915$
MM	$\bar{\lambda} = 4.061$ $\sigma_\lambda = 0.323$ $E_\lambda = 0.262$	$\bar{\mu} = 99.195$ $\sigma_\mu = 7.255$ $E_\mu = 6.001$	$\bar{\lambda} = 7.543$ $\sigma_\lambda = 0.539$ $E_\lambda = 0.479$	$\bar{\mu} = 99.896$ $\sigma_\mu = 6.947$ $E_\mu = 6.044$
$M_\gamma$	$\bar{\lambda} = 4.058$ $\sigma_\lambda = 0.302$ $E_\lambda = 0.251$	$\bar{\mu} = 99.176$ $\sigma_\mu = 6.729$ $E_\mu = 5.806$	$\bar{\lambda} = 7.599$ $\sigma_\lambda = 0.554$ $E_\lambda = 0.504$	$\bar{\mu} = 99.188$ $\sigma_\mu = 7.042$ $E_\mu = 6.309$
MN	$\bar{\lambda} = 4.148$ $\sigma_\lambda = 0.310$	$\bar{\mu} = 98.343$ $\sigma_\mu = 7.023$	$\bar{\lambda} = 7.603$ $\sigma_\lambda = 0.542$	$\bar{\mu} = 99.529$ $\sigma_\mu = 6.888$

TABLE II  
COMPARISON OF THE ESTIMATES OF RICIAN PARAMETERS FOR HIGHER  $\lambda$  VALUES WITH THE METHODS: ML, MM,  $M_\gamma$  AND MN (GAUSSIAN APPROXIMATION) (128 REPEATS). THE PARAMETERS ARE  $\mu = 100$ . AND  $\lambda = 4$  AND 7.5. THE NUMBER OF SAMPLES IS 100. FOR THE ML METHOD, TO REACH CONVERGENCE 13 ITERATIONS ARE NECESSARY FOR BOTH  $\lambda$  VALUES.

requires a unique estimation of the first two moments and less than twenty arithmetical operations. Finally, the  $M_\gamma$  method is 370 times faster than the ML approach in the case of low  $\lambda$  values, and 7000 times faster in the case of high  $\lambda$  values.

#### D. A link between Rician distribution and Ferreti criterion

In their famous paper about permanent scatterers, Ferreti et al. [15] deal with the *Dispersion index*  $D_A$  :

$$D_A = \frac{\sigma_A}{m_A}, \quad (25)$$

where  $m_A$  and  $\sigma_A$  are the mean and the standard deviation of the amplitude values corresponding to a given point of a temporal stack. Actually, this *Dispersion index* can be seen as the coefficient of variation of the Rician distribution associated to the target. Targets can be seen as permanent scatterers if they exhibit  $D_A$  values under a given threshold (typically  $D_A < 0.25$ ). With the help of equation (15), we have the following relation :

$$D_A < 0.25 \Leftrightarrow \lambda > 2.679, \quad (26)$$

so that a Rician distribution corresponding to a target defined by its value  $\lambda$  can be viewed as a Permanent Scatterer if  $\lambda > 2.679$ , meaning that the target is 2.7 higher than the surrounding speckle.

#### E. Preliminary experiments

To illustrate the interest of the temporal Rice distribution, we present the temporal histogram of a permanent scatterer point (figure 5). The  $\lambda$  value is 2.81 ( $D_A = 0.241$ ) and the number of used dates is 134 (Fos oil terminal SLC Sentinel-1 images, France).

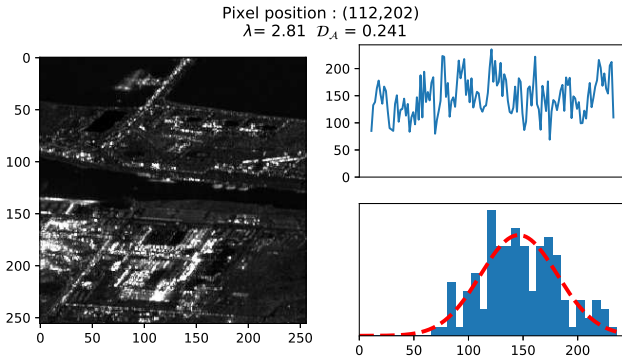


Fig. 5. Temporal histogram for a permanent point. On the left the average stack of the images with the permanent scatterer located in (112,202). On the right, the temporal evolution of the point and its histogram showing a Rice behaviour (134 temporal samples are used).

The following points have to be addressed in the future. First, the number of samples remains limited even with very long time series of 134 images. Nevertheless it is not possible to use spatial samples to increase the number of samples, since the surrounding pixels will probably not contain the same dominant phasor. Besides, the temporal correlation between the samples should be taken into account. The last point to be investigated is the phase information after the correction of the interferometric phase contribution.

## IV. CONCLUSION

In this paper we have proposed a new parametrization for the Rician distribution and introduced a new parameter the relative drift (ratio of target backscattering to speckle) to characterize the pdf. We also proposed an efficient numeric estimation of this parameter through the coefficient of variation. Different estimation methods for Rician pdf parameters have been presented and compared, showing the RPM approximation based on the coefficient of variation is a good compromise between the variance of the estimate and the computation time. Furthermore, the link between permanent scatterer criterion and this new parameter has been established.

The proposed parametrization and associated parameter estimation method can be useful for many applications relying on the Rician distribution. In particular, further work will be dedicated to the exploitation of this parametrization to detect and characterize strong targets in long temporal SAR series. Since for long-time series the  $\mu$  contribution is rarely constant, further work will be dedicated to define strategies allowing to select samples with similar  $\mu$  values.

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