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Guessing a Secret Cryptographic Key from Side-Channel Leakages

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Abstract

We experiment relative merits of information-theoretic metrics such as guessing entropy, conditional Shannon or Rényi entropies vs. success probability, in the problem of guessing a cryptographic key form a leakage in some practical cryptosystems, with Hamming weight leakage model in additive (Gaussian) measurement noise.

This is ongoing work with Sylvain Guilley (Telecom Paris, Secure-IC) and Olivier Rioul (Telecom Paris)

Keywords. Guessing entropy, Conditional Shannon entropy, Rényi entropy, Success probability

Information-theoretic metrics







(10)

(11)

Figure 1: Leakage model: secret X, noise Z and leakage Y

Let X be a *discrete* random variable with probability distribution p(x). Without loss of generality we may suppose that $X \in \{1, 2, ..., n, ...\}$ with respective probabilities $p_1, p_2, ..., p_n, ...$ Let Y = f(X) + Z be additional information (*leakage*) about X. If noise Z is present, Y is a continuous r.v. with density p(y), while in the noiseless case (Z = 0), Y is discrete with distribution p(y). The attacker knows Y and guesses X. We have the following metrics:

• (Conditional) Guessing entropy: letting $p_k = p(x = k), k = 1, 2, ..., n, ...,$ we have the (conditional) guessing entropies G(X) and G(X|Y) as:

$$G(X) = \sum_{k} k p_{(k)}, \qquad G(X|Y) = \oint p(y)G(X|Y=y)$$
(1)

(2)

(5)

(8)

(9)

where the probabilities are arranged in decreasing order $p_{(1)} \ge p_{(2)} \ge \cdots \ge p_{(n)} \ge \cdots$.

• (Conditional) Shannon Entropies:

$$H(X) = \sum_{x \in \mathcal{X}} p(x) \log_2 \frac{1}{p(x)}$$
$$H(X|Y) = \oint_{y \in \mathcal{Y}} p(y) \sum_{x \in \mathcal{X}} p(x|y) \log \frac{1}{p(x|y)}$$

• (Conditional) Arimoto-Rényi Entropies:

$$H_{\alpha}(X) = \frac{\alpha}{1-\alpha} \log \left(\sum_{x} p(x)^{\alpha} \right)^{1/\alpha}$$

$$H_{\alpha}(X|Y) = \frac{\alpha}{1-\alpha} \log \oint_{y \in \mathcal{Y}} p(y) \left(\sum_{x} p(x|y)^{\alpha} \right)^{1/\alpha}$$
(3)

• (Conditional) Success probability:

Figure 2: Conditional Shannon and Rényi Entropies of X with Hamming weight leakages

Guessing X with Noisy Hamming Weight Leakages

In fact, noise is the intrinsic part in the side-channel leakages, like power consumption and electromagnetic radiations. Thus we consider the noisy leakages in a classic way by assuming the noise is the additive white Gaussian noise (AWGN), which is a basic noise model to mimic the effect of many random processes. We assume that $Z \sim \mathcal{N}(0, \sigma^2)$ of standard normal density $\varphi(z)$ which is a nonincreasing function of |z|. Thus we have:

$$p(x) = \frac{1}{M}, \qquad p(y) = \sum_{x} p(x)p(y|x) = \frac{1}{M}\sum_{x} \varphi(y - f(x))$$

$$p(y|x) = \varphi(y - f(x)), \qquad p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \frac{\varphi(y)}{\sum_{x'} \varphi(y - f(x'))}$$

In addition, maximum conditional probability of success is computed as follows.

$$P_{s}(X|Y) = \mathbb{E}\max_{x} p(x|Y) = \int \left(\frac{1}{M} \sum_{x'} \varphi(y - f(x'))\right) \times \frac{\varphi(\min_{x} |y - f(x)|)}{\sum_{x'} \varphi(y - f(x'))} dy$$
$$= \frac{1}{M} \int \varphi(y - f(x^{*}(y))) dy \quad (\text{where } x^{*}(y) = \arg\min_{x} |y - f(x)|)$$
$$= \frac{M'}{M} - 2\frac{M' - 1}{M} Q\left(\frac{\Delta/2}{\sigma}\right) \qquad (\text{where } Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right))$$
$$H(X|Y) = H(X) - h(Y) + h(Y|X) = \log M + \frac{1}{2} \log(2\pi e\sigma^{2}) - \int p(y) \log\frac{1}{p(y)} dy$$

$$P_s(X) = \max_x p(x), \qquad P_s(X|Y) = \sum_{y \in \mathcal{Y}} p(y) \max_x p(x|y) \ge P_s(X)$$
(4)

Guessing X with Noiseless Hamming Weight Leakages

Hamming weight leakage model $f = w_H$ is one of the most general leakage model used in side-channel analysis. Particularly, hardware implementations leak bits in parallel, hence the leakage is the sum of the registers state bits, that is the Hamming weight of the register contents.

Let $Y = w_H(X)$ where w_H is the Hamming weight function, in the noiseless case (Z = 0). We choose $|\mathcal{X}| = M = 2^n$ for the sake of calculation.

$$p(x) = \frac{1}{2^n}, \qquad p(y) = \frac{\binom{n}{y}}{2^n}, \qquad p(x|y) = \frac{\mathbf{1}_{y=w_H(x)}}{\binom{n}{y}}$$

We focus on quantifying the reduction of uncertainty of X knowing Y. Thus,

• (Conditional) Guessing entropy:

$$G(X) = \sum_{k} p_{k} = \sum_{k=1}^{2^{n}} k \cdot \frac{1}{2^{n}} = \frac{2^{n} + 1}{2}$$

$$G(X|Y) = \sum_{y} \mathbb{P}(y) \sum_{x} x \cdot \mathbb{P}(x|y) = \frac{1}{2} + \frac{1}{2^{n+1}} \binom{2n}{n} \approx \frac{1}{2} \left(1 + \frac{2^{n}}{\sqrt{\pi n}} \right)$$
(6)

• (Conditional) Shannon Entropies:

$$H(X) = \sum_{x} p(x) \log \frac{1}{p(x)} = \log 2^{n} = n$$

$$H(X|Y) = -\sum_{x,y} p(x,y) \log p(x|y) = 2^{-n} \sum_{y} \binom{n}{y} \cdot \log \binom{n}{y}$$
(7)

• Conditional Rényi Entropies:

Numerical Comparison with Lower and Upper Bounds of G(X|Y)

We present here six upper and lower bounds of guessing entropy of X by knowing its Hamming weight leakages. Interestingly, Bostas's upper bound is the best one which is identical to guessing entropy, which in the Hamming weight leakage scenarios.



Figure 3: Comparison of six upper and lower bounds of G(X)

Preliminary Conclusions

We present two scenarios of guessing a secret X with Hamming weight leakages. Specifically, with small $M = 2^n$, this type of leakage has much more impact on the conditional entropies, which are the common cases in embedded systems. This explains why the Divide-and-Conquer attacks work in side-channel analysis. However, with large M, such as $M = 2^{128}$ for the AES-128 cryptographic key, the Hamming weight of

$$H_{\alpha}(X|Y) = \frac{\alpha}{1-\alpha} \log \sum_{y} p(y) \left(\sum_{x} p(x|y)^{\alpha} \right)^{\frac{1}{\alpha}} = \frac{\alpha}{\alpha-1} \left(n - \log \sum_{y} \binom{n}{y}^{\frac{1}{\alpha}} \right)$$

• Conditional Success probability:

 $\alpha < 1.0.$

$$P_s(X|Y) = \mathbb{E}_Y \max_x p(x|Y) = \frac{M'}{M} = \frac{n+1}{2^n}$$

Numerical Results on Noiseless Leakages

By upper bound from Fano's inequality and lower bound $H(X|Y) \geq \varphi^*(P_s(X|Y))$ where $\varphi^*(s) =$ $\left\lfloor \frac{1}{s} \right\rfloor \left(s \left\lceil \frac{1}{s} \right\rceil - 1 \right) \log \left\lfloor \frac{1}{s} \right\rfloor + \left(1 - \left\lfloor \frac{1}{s} \right\rfloor \left(s \left\lceil \frac{1}{s} \right\rceil - 1 \right) \right) \log \left\lceil \frac{1}{s} \right\rceil \text{ and } H_{\alpha}(X|Y) \ge \frac{\alpha}{1 - \alpha} \log \phi_{\alpha}^{*}(P_{s}(X|Y)), \text{ where } \phi_{\alpha}^{*}(s) = \frac{\alpha}{1 - \alpha} \log \phi_{\alpha}^{*}(S)$ $\left(\left\lceil\frac{1}{s}\right\rceil s - 1\right)\left\lfloor\frac{1}{s}\right\rfloor^{1/\alpha} + \left(1 - \left\lfloor\frac{1}{s}\right\rfloor\left(\left\lceil\frac{1}{s}\right\rceil s - 1\right)\right)\left\lceil\frac{1}{s}\right\rceil^{\frac{1-\alpha}{\alpha}}$ (by Sason et al. [1]), we numerically show the conditional Shannon and Rényi entropies of X as Fig. 2. Specifically, the upper bound of Rényi entropy is highly dependent on the α . With α much larger than 1.0, the marked region is much smaller than the region with

whole key is of very little help for the attacker.

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