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# Guessing a Secret Cryptographic Key from Side-Channel Leakages

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## Abstract

We experiment relative merits of information-theoretic metrics such as guessing entropy, conditional Shannon or Rényi entropies vs. success probability, in the problem of guessing a cryptographic key from a leakage in some practical cryptosystems, with Hamming weight leakage model in additive (Gaussian) measurement noise.

This is ongoing work with Sylvain Guilley (Telecom Paris, Secure-IC) and Olivier Rioul (Telecom Paris)

**Keywords.** Guessing entropy, Conditional Shannon entropy, Rényi entropy, Success probability

## Information-theoretic metrics

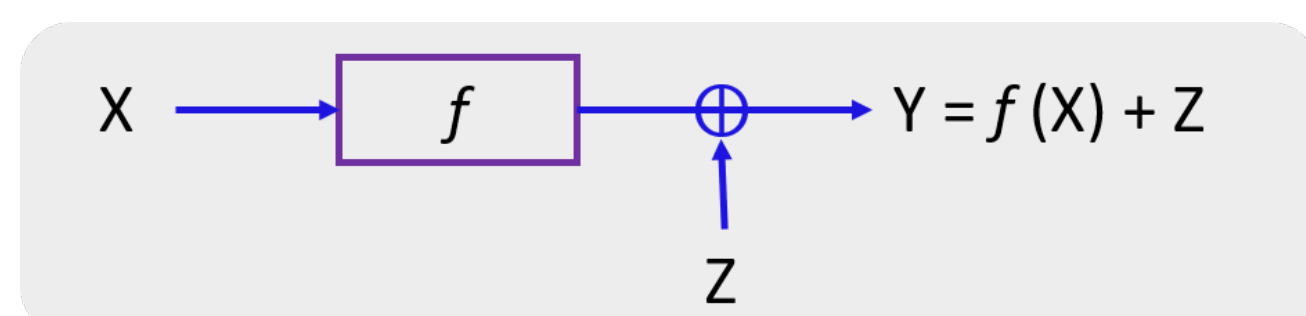


Figure 1: Leakage model: secret  $X$ , noise  $Z$  and leakage  $Y$

Let  $X$  be a discrete random variable with probability distribution  $p(x)$ . Without loss of generality we may suppose that  $X \in \{1, 2, \dots, n, \dots\}$  with respective probabilities  $p_1, p_2, \dots, p_n, \dots$ . Let  $Y = f(X) + Z$  be additional information (leakage) about  $X$ . If noise  $Z$  is present,  $Y$  is a continuous r.v. with density  $p(y)$ , while in the noiseless case ( $Z = 0$ ),  $Y$  is discrete with distribution  $p(y)$ . The attacker knows  $Y$  and guesses  $X$ . We have the following metrics:

- **(Conditional) Guessing entropy:** letting  $p_k = p(x = k)$ ,  $k = 1, 2, \dots, n, \dots$ , we have the (conditional) guessing entropies  $G(X)$  and  $G(X|Y)$  as:

$$G(X) = \sum_k k p_k, \quad G(X|Y) = \int p(y) G(X|Y=y) \quad (1)$$

where the probabilities are arranged in decreasing order  $p_{(1)} \geq p_{(2)} \geq \dots \geq p_{(n)} \geq \dots$ .

- **(Conditional) Shannon Entropies:**

$$H(X) = \sum_{x \in \mathcal{X}} p(x) \log_2 \frac{1}{p(x)} \quad (2)$$

$$H(X|Y) = \int_{y \in \mathcal{Y}} p(y) \sum_{x \in \mathcal{X}} p(x|y) \log \frac{1}{p(x|y)}$$

- **(Conditional) Arimoto-Rényi Entropies:**

$$H_\alpha(X) = \frac{\alpha}{1-\alpha} \log \left( \sum_x p(x)^\alpha \right)^{1/\alpha} \quad (3)$$

$$H_\alpha(X|Y) = \frac{\alpha}{1-\alpha} \log \int_{y \in \mathcal{Y}} p(y) \left( \sum_x p(x|y)^\alpha \right)^{1/\alpha}$$

- **(Conditional) Success probability:**

$$P_s(X) = \max_x p(x), \quad P_s(X|Y) = \int_{y \in \mathcal{Y}} p(y) \max_x p(x|y) \geq P_s(X) \quad (4)$$

## Guessing $X$ with Noiseless Hamming Weight Leakages

Hamming weight leakage model  $f = w_H$  is one of the most general leakage model used in side-channel analysis. Particularly, hardware implementations leak bits in parallel, hence the leakage is the sum of the registers state bits, that is the Hamming weight of the register contents.

Let  $Y = w_H(X)$  where  $w_H$  is the Hamming weight function, in the noiseless case ( $Z = 0$ ). We choose  $|\mathcal{X}| = M = 2^n$  for the sake of calculation.

$$p(x) = \frac{1}{2^n}, \quad p(y) = \frac{\binom{n}{y}}{2^n}, \quad p(x|y) = \frac{\mathbf{1}_{y=w_H(x)}}{\binom{n}{y}} \quad (5)$$

We focus on quantifying the reduction of uncertainty of  $X$  knowing  $Y$ . Thus,

- **(Conditional) Guessing entropy:**

$$G(X) = \sum_k p_k = \sum_{k=1}^{2^n} k \cdot \frac{1}{2^n} = \frac{2^n + 1}{2} \quad (6)$$

$$G(X|Y) = \sum_y \mathbb{P}(y) \sum_x x \cdot \mathbb{P}(x|y) = \frac{1}{2} + \frac{1}{2^{n+1}} \binom{2n}{n} \approx \frac{1}{2} \left( 1 + \frac{2^n}{\sqrt{\pi n}} \right)$$

- **(Conditional) Shannon Entropies:**

$$H(X) = \sum_x p(x) \log \frac{1}{p(x)} = \log 2^n = n \quad (7)$$

$$H(X|Y) = - \sum_{x,y} p(x,y) \log p(x|y) = 2^{-n} \sum_y \binom{n}{y} \cdot \log \binom{n}{y}$$

- **Conditional Rényi Entropies:**

$$H_\alpha(X|Y) = \frac{\alpha}{1-\alpha} \log \sum_y p(y) \left( \sum_x p(x|y)^\alpha \right)^{\frac{1}{\alpha}} = \frac{\alpha}{\alpha-1} \left( n - \log \sum_y \binom{n}{y}^\alpha \right) \quad (8)$$

- **Conditional Success probability:**

$$P_s(X|Y) = \mathbb{E}_Y \max_x p(x|Y) = \frac{M'}{M} = \frac{n+1}{2^n} \quad (9)$$

## Numerical Results on Noiseless Leakages

By upper bound from Fano's inequality and lower bound  $H(X|Y) \geq \varphi^*(P_s(X|Y))$  where  $\varphi^*(s) = \left\lfloor \frac{1}{s} \right\rfloor (s \left\lfloor \frac{1}{s} \right\rfloor - 1) \log \left\lfloor \frac{1}{s} \right\rfloor + \left( 1 - \left\lfloor \frac{1}{s} \right\rfloor (s \left\lfloor \frac{1}{s} \right\rfloor - 1) \right) \log \left\lceil \frac{1}{s} \right\rceil$  and  $H_\alpha(X|Y) \geq \frac{\alpha}{1-\alpha} \log \varphi_\alpha^*(P_s(X|Y))$ , where  $\varphi_\alpha^*(s) = \left( \left\lfloor \frac{1}{s} \right\rfloor s - 1 \right) \left\lfloor \frac{1}{s} \right\rfloor^{1/\alpha} + \left( 1 - \left\lfloor \frac{1}{s} \right\rfloor (s \left\lfloor \frac{1}{s} \right\rfloor - 1) \right) \left\lceil \frac{1}{s} \right\rceil^{1/\alpha}$  (by Sason et al. [1]), we numerically show the conditional Shannon and Rényi entropies of  $X$  as Fig. 2. Specifically, the upper bound of Rényi entropy is highly dependent on the  $\alpha$ . With  $\alpha$  much larger than 1.0, the marked region is much smaller than the region with  $\alpha < 1.0$ .

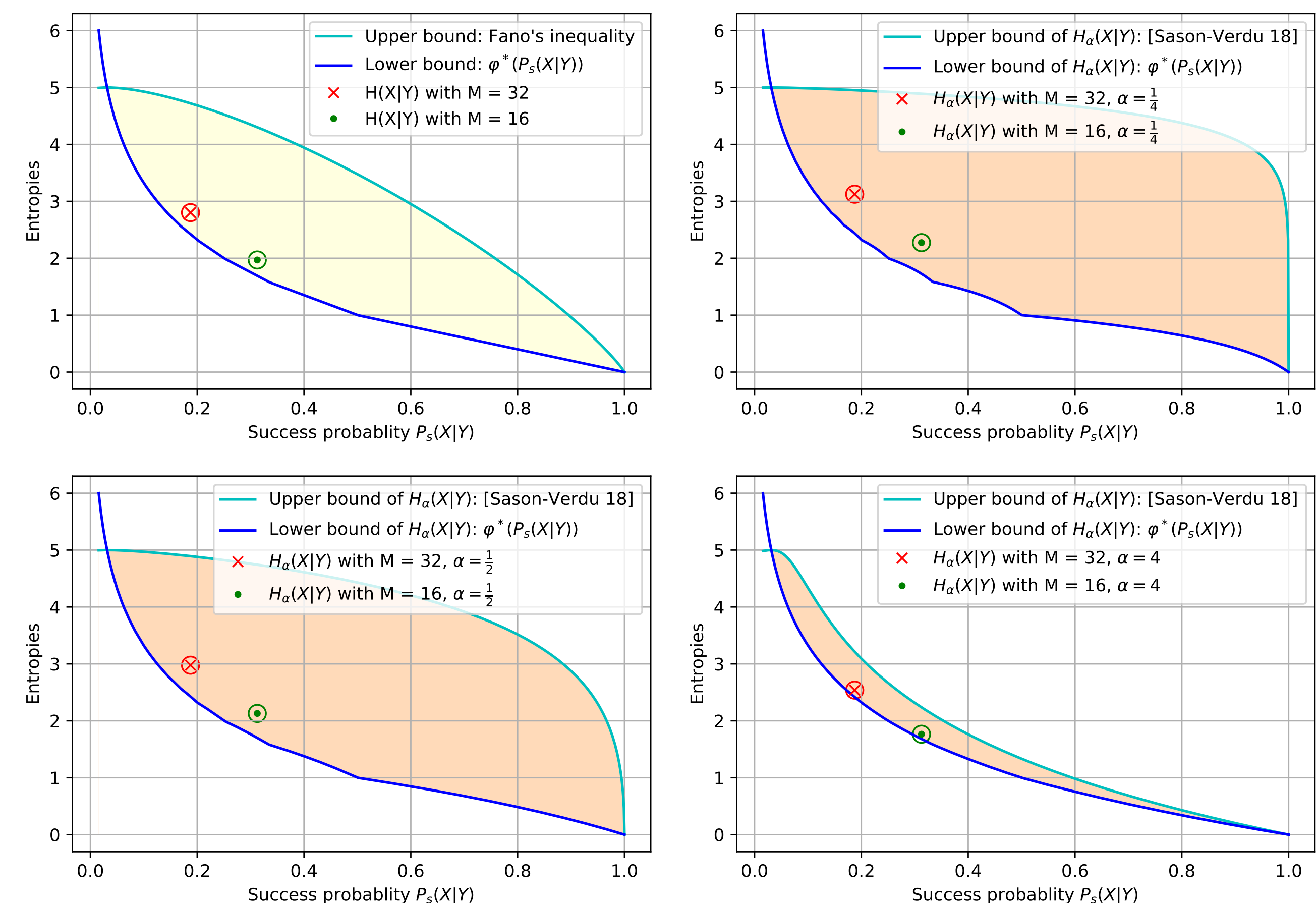


Figure 2: Conditional Shannon and Rényi Entropies of  $X$  with Hamming weight leakages

## Guessing $X$ with Noisy Hamming Weight Leakages

In fact, noise is the intrinsic part in the side-channel leakages, like power consumption and electromagnetic radiations. Thus we consider the noisy leakages in a classic way by assuming the noise is the additive white Gaussian noise (AWGN), which is a basic noise model to mimic the effect of many random processes.

We assume that  $Z \sim \mathcal{N}(0, \sigma^2)$  of standard normal density  $\varphi(z)$  which is a nonincreasing function of  $|z|$ . Thus we have:

$$p(x) = \frac{1}{M}, \quad p(y) = \sum_x p(x) p(y|x) = \frac{1}{M} \sum_x \varphi(y - f(x)) \quad (10)$$

$$p(y|x) = \varphi(y - f(x)), \quad p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \frac{\varphi(y - f(x))}{\sum_{x'} \varphi(y - f(x'))}$$

In addition, maximum conditional probability of success is computed as follows.

$$P_s(X|Y) = \mathbb{E} \max_x p(x|Y) = \int \left( \frac{1}{M} \sum_{x'} \varphi(y - f(x')) \right) \times \frac{\varphi(\min_x |y - f(x)|)}{\sum_{x'} \varphi(y - f(x'))} dy$$

$$= \frac{1}{M} \int \varphi(y - f(x^*(y))) dy \quad (\text{where } x^*(y) = \arg \min_x |y - f(x)|) \quad (11)$$

$$= \frac{M'}{M} - 2 \frac{M' - 1}{M} Q \left( \frac{\Delta/2}{\sigma} \right) \quad (\text{where } Q(x) = \frac{1}{2} \text{erfc} \left( \frac{x}{\sqrt{2}} \right))$$

$$H(X|Y) = H(X) - h(Y) + h(Y|X) = \log M + \frac{1}{2} \log(2\pi\sigma^2) - \int p(y) \log \frac{1}{p(y)} dy$$

## Numerical Comparison with Lower and Upper Bounds of $G(X|Y)$

We present here six upper and lower bounds of guessing entropy of  $X$  by knowing its Hamming weight leakages. Interestingly, Bostas's upper bound is the best one which is identical to guessing entropy, which in the Hamming weight leakage scenarios.

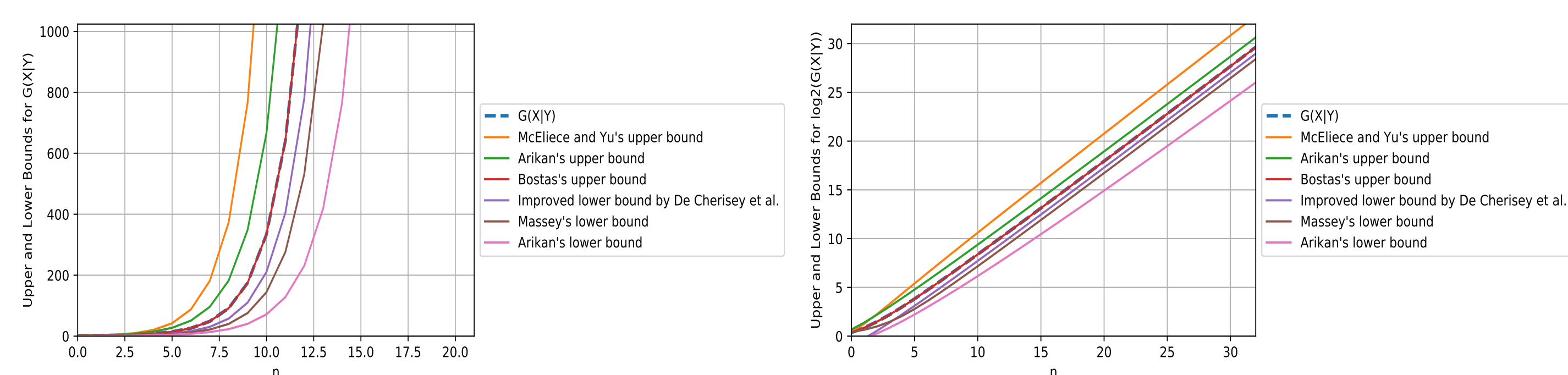


Figure 3: Comparison of six upper and lower bounds of  $G(X|Y)$

## Preliminary Conclusions

We present two scenarios of guessing a secret  $X$  with Hamming weight leakages. Specifically, with small  $M = 2^n$ , this type of leakage has much more impact on the conditional entropies, which are the common cases in embedded systems. This explains why the Divide-and-Conquer attacks work in side-channel analysis. However, with large  $M$ , such as  $M = 2^{128}$  for the AES-128 cryptographic key, the Hamming weight of whole key is of very little help for the attacker.

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