



HAL
open science

Optimal attacks for multivariate and multi-model side-channel leakages

Nicolas Bruneau, Sylvain Guilley, Annelie Heuser, Marion Damien, Olivier Rioul

► **To cite this version:**

Nicolas Bruneau, Sylvain Guilley, Annelie Heuser, Marion Damien, Olivier Rioul. Optimal attacks for multivariate and multi-model side-channel leakages. 18th Workshop on Cryptographic Hardware and Embedded Systems (CHES 2016), Aug 2016, Santa Barbara, United States. Cryptographic Hardware and Embedded Systems – CHES 2016, 2016. hal-02300058

HAL Id: hal-02300058

<https://telecom-paris.hal.science/hal-02300058>

Submitted on 20 Aug 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Optimal Attacks for Multivariate and Multi-model Side-Channel Leakages

Nicolas Bruneau, Sylvain Guilley, Annelie Heuser, Damien Marion and Olivier Rioul
firstname.lastname@telecom-paristech.fr



Abstract. In practice, a side-channel signal is measured as a trace consisting of *several* samples where *several* sensitive bits are manipulated in parallel, each leaking differently. Therefore, the informed attacker needs to devise side-channel distinguishers that can handle both *multivariate* leakages and *multivariate* models at the same time. In the state of the art, these two issues have two independent solutions: on the one hand, dimensionality reduction can cope with multivariate leakage; on the other hand, online stochastic approaches can cope with multivariate models.

In this work, we combine both solutions to derive closed-form expressions of the resulting *optimal* distinguisher in terms of matrix operations, in all situations where the model can be either profiled offline or regressed online. Optimality here means that the success probability is maximized for a given number of traces. We recover known results for uni- and bi-variate models (including correlation power analysis), and investigate novel distinguishers for multivariate models with more than two parameters. Following ideas from the AsiaCrypt'2013 paper "*Behind the Scene of Side-Channel Attacks*", we also provide fast computation algorithms in which the traces are accumulated prior to computing the distinguisher values.

(full version at PROOFS 2016)

1. Fact

Side-channel leakages are:

- **multi-variate** (in time)
- **multi-model** (e.g., each bit leaks ≠)

2. Matrix Notations

- Q number of queries,
- D number of samples,
- S number of models.

In matrix notation:

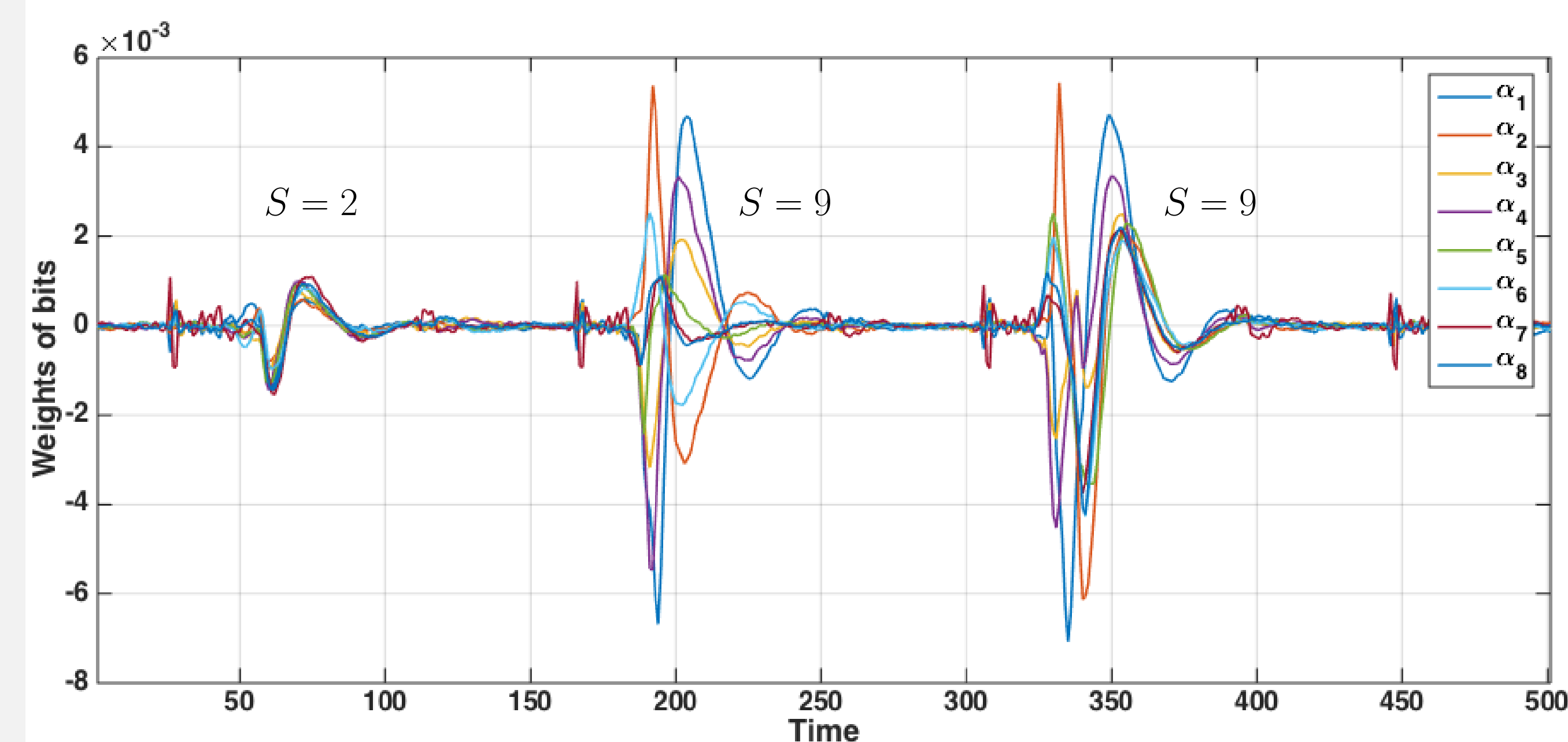
$$\mathbf{X} = \alpha \mathbf{Y}^* + \mathbf{N} \quad (1)$$

where

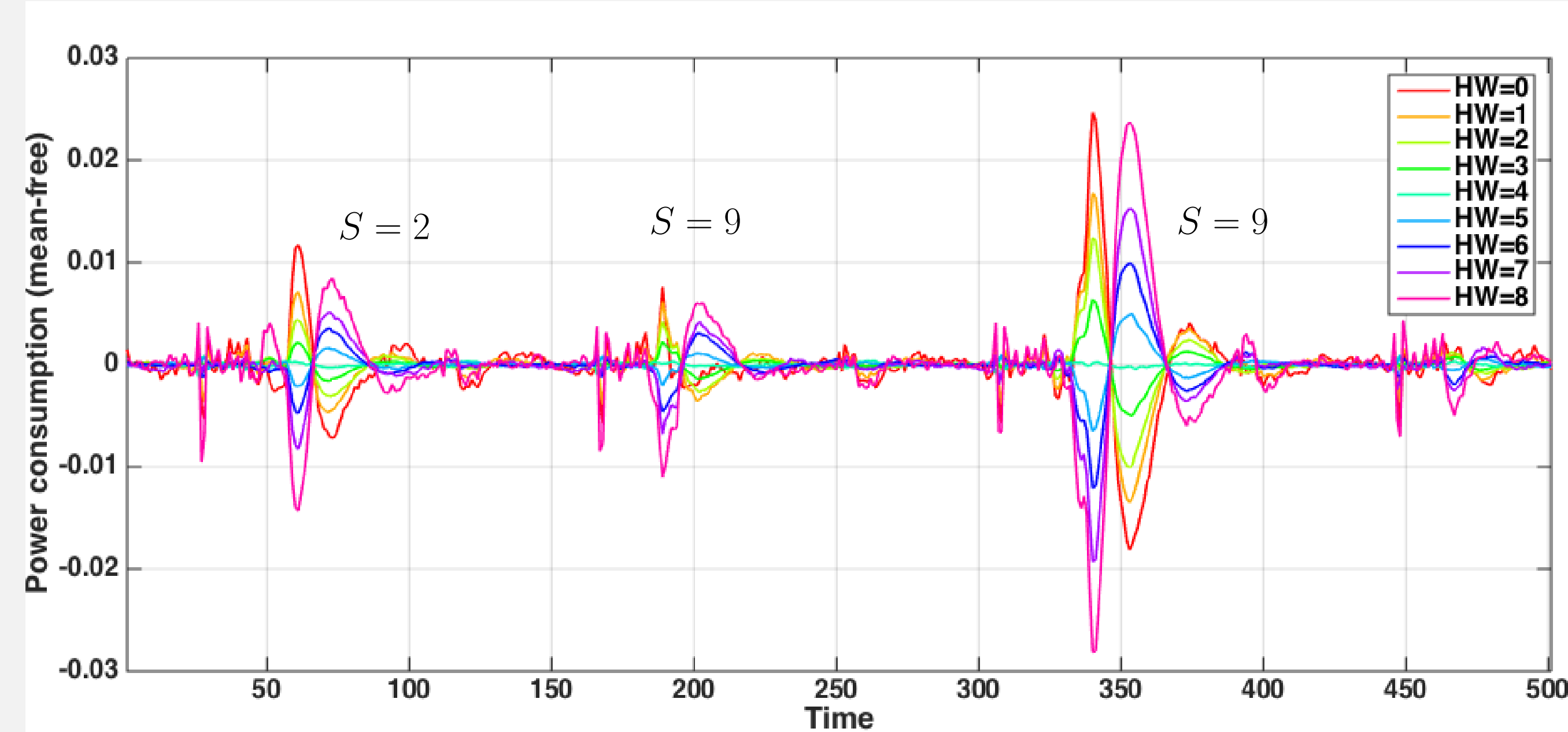
- \mathbf{X} is a matrix of size $D \times Q$,
- α is a matrix of size $D \times S$,
- \mathbf{Y}^* (the star means: "for the correct key $k = k^*$ ") is a matrix of size $S \times Q$,
- \mathbf{N} is a matrix of size $D \times Q$.

3. Real World Example

The figures below show power consumption traces taken from an ATmega smartcard—datasets are available from the DPA contest V4 team [?] (knowing the mask).



(a) Weights of bits of the sensitive variable



(b) Mean power consumption for each Hamming weight class

4. Question

What is the optimal distinguisher, when in Equation (1):

- α is known? $\mathcal{D}_{ML}(\mathbf{x}, \mathbf{t})$
- α is unknown? $\mathcal{D}_{ML,sto}(\mathbf{x}, \mathbf{t})$

5. Solution

Theorem 1. The optimal maximum likelihood (ML) distinguisher [?] for Gaussian noise writes

$$\mathcal{D}_{ML}(\mathbf{x}, \mathbf{t}) = \operatorname{argmin}_k \operatorname{tr} \left((\mathbf{x} - \alpha \mathbf{y})^\top \Sigma^{-1} (\mathbf{x} - \alpha \mathbf{y}) \right). \quad (2)$$

Proof. From [?] we have $\mathcal{D}_{ML}(\mathbf{x}, \mathbf{t}) = \operatorname{argmax}_k p(\mathbf{x}|\mathbf{y})$ where from (1) it is easily seen that $p(\mathbf{x}|\mathbf{y}) = p_{\mathbf{N}}(\mathbf{x} - \alpha \mathbf{y})$. From the i.i.d. assumption the noise density $p_{\mathbf{N}}(\mathbf{n})$ is given by

$$p_{\mathbf{N}}(\mathbf{n}) = \prod_{q=1}^Q \frac{1}{\sqrt{(2\pi)^D |\det \Sigma|}} \exp \left(-\frac{1}{2} n_q^\top \Sigma^{-1} n_q \right) \quad (3)$$

$$= \frac{1}{(2\pi)^{DQ/2} (\det \Sigma)^{Q/2}} \exp \left(-\frac{1}{2} \left(\sum_{q=1}^Q n_q^\top \Sigma^{-1} n_q \right) \right) \quad (4)$$

$$= \frac{1}{(2\pi)^{DQ/2} (\det \Sigma)^{Q/2}} \exp \left(-\frac{1}{2} \operatorname{tr} (\mathbf{n}^\top \Sigma^{-1} \mathbf{n}) \right). \quad (5)$$

Thus $p_{\mathbf{N}}(\mathbf{x} - \alpha \mathbf{y})$ is maximum when the expression $\operatorname{tr} (\mathbf{n}^\top \Sigma^{-1} \mathbf{n})$ for $\mathbf{n} = \mathbf{x} - \alpha \mathbf{y}$ is minimum. \square

Theorem 2. The optimal stochastic multivariate attack is given by

$$\mathcal{D}_{ML,sto}(\mathbf{x}, \mathbf{t}) = \operatorname{argmax}_{k \in \mathbb{F}_2^S} \operatorname{tr} \left(\mathbf{y}^\top (\mathbf{y} \mathbf{y}^\top)^{-1} \mathbf{y} \mathbf{x}^\top \Sigma^{-1} \mathbf{x} \right). \quad (6)$$

for which the optimal value of α is given by

$$\alpha^{opt} = (\mathbf{x} \mathbf{y}^\top) (\mathbf{y} \mathbf{y}^\top)^{-1}. \quad (7)$$

Proof. Let $\mathbf{x}' = \Sigma^{-1/2} \mathbf{x}$ and $\mathbf{y}' = (\mathbf{y} \mathbf{y}^\top)^{-1/2} \mathbf{y}$. The optimal distinguisher minimizes the following expression over $\alpha \in \mathbb{R}^{D \times S}$:

$$\operatorname{tr} \left((\mathbf{x} - \alpha \mathbf{y})^\top \Sigma^{-1} (\mathbf{x} - \alpha \mathbf{y}) \right) = \operatorname{tr} \left((\mathbf{x}' - \alpha' \mathbf{y}')^\top (\mathbf{x}' - \alpha' \mathbf{y}') \right) = \sum_{d=1}^D \|\mathbf{x}'_d - \alpha'_d \mathbf{y}'_d\|^2.$$

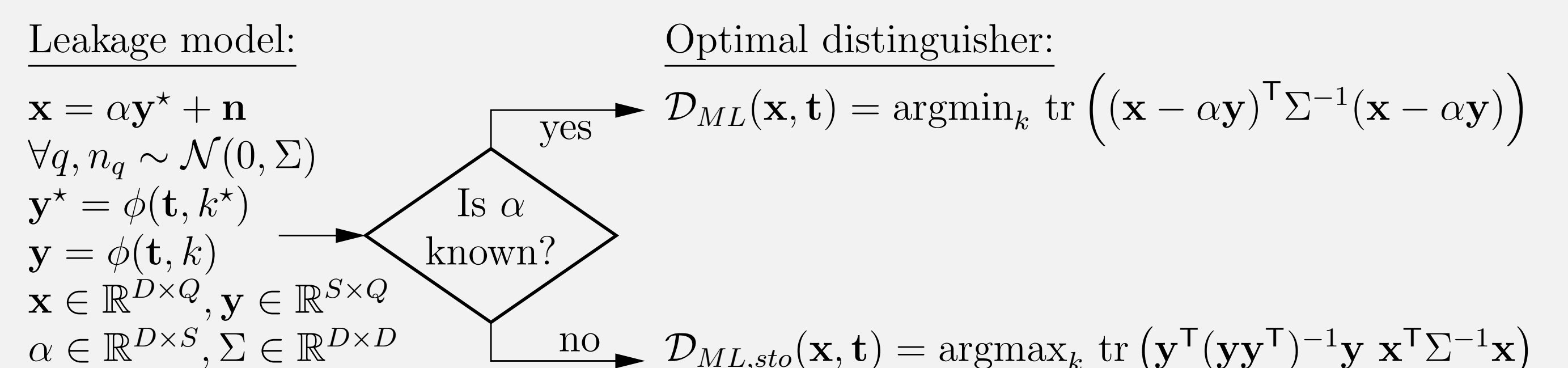
The minimization over α'_d yields $\alpha'_d = (\mathbf{x}'_d \mathbf{y}'_d{}^\top) (\mathbf{y}'_d \mathbf{y}'_d{}^\top)^{-1}$ for all $d = 1, \dots, D$. This gives $\alpha' = (\mathbf{x}' \mathbf{y}'^\top) (\mathbf{y}' \mathbf{y}'^\top)^{-1}$ hence $\alpha = (\mathbf{x} \mathbf{y}^\top) (\mathbf{y} \mathbf{y}^\top)^{-1}$, which remarkably does *not* depend on Σ . The minimized value of the distinguisher is thus

$$\begin{aligned} \min_{\alpha} \operatorname{tr} \left((\mathbf{x} - \alpha \mathbf{y})^\top \Sigma^{-1} (\mathbf{x} - \alpha \mathbf{y}) \right) &= \operatorname{tr} \left((\mathbf{x} - \alpha^{opt} \mathbf{y})^\top \Sigma^{-1} (\mathbf{x} - \alpha^{opt} \mathbf{y}) \right) \\ &= \operatorname{tr} \left((\mathbf{I} - \mathbf{y}^\top (\mathbf{y} \mathbf{y}^\top)^{-1} \mathbf{y}) \mathbf{x}^\top \Sigma^{-1} \mathbf{x} \right) \\ &= \operatorname{tr} (\mathbf{x}^\top \Sigma^{-1} \mathbf{x}) - \operatorname{tr} \left(\mathbf{y}^\top (\mathbf{y} \mathbf{y}^\top)^{-1} \mathbf{x}^\top \Sigma^{-1} \mathbf{x} \right) \end{aligned}$$

where \mathbf{I} is the $D \times D$ identity matrix and where $\operatorname{tr} (\mathbf{x}^\top \Sigma^{-1} \mathbf{x})$ is a constant independent of k . \square

6. Summary for $S > 2$ Models

Mathematical expression for multivariate ($D \geq 1$) optimal attacks with a linear combination of models ($S \geq 1$):



6bis. Summary for $S = 2$ Models

Modus operandi for multivariate ($D \geq 1$) optimal attacks with one model \mathbf{Y} associated to envelope $\alpha \in \mathbb{R}^{D \times 1}$ and a constant offset $\beta \in \mathbb{R}^{D \times 1}$ ($S = 2$):

