Optimal attacks for multivariate and multi-model side-channel leakages
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To cite this version:
Nicolas Bruneau, Sylvain Guilley, Annelie Heuser, Marion Damien, Olivier Rioul. Optimal attacks for multivariate and multi-model side-channel leakages. 18th Workshop on Cryptographic Hardware and Embedded Systems (CHES 2016), Aug 2016, Santa Barbara, United States. Cryptographic Hardware and Embedded Systems – CHES 2016, 2016. hal-02300058

HAL Id: hal-02300058
https://telecom-paris.hal.science/hal-02300058
Submitted on 20 Aug 2021

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Abstract. In practice, a side-channel signal is measured as a trace consisting of several samples where several sensitive bits are manipulated in parallel, each leaking differently. Therefore, the informed attacker needs to devise side-channel distinguishers that can handle both multivariate leaks and multivariate models at the same time. In the state of the art, these two issues have two independent solutions: on the one hand, dimensionality reduction can cope with multivariate leakage; on the other hand, online stochastic approaches can cope with multivariate models.

In this work, we combine both solutions to derive closed-form expressions of the optimal distinguisher in terms of matrix operations, in all situations where the model can be either profiled offline or regressed online. Optimality here means that the success probability is maximized for a given number of traces. We recover known results for uni- and bi-variate models (including correlation power analysis), and investigate novel distinguishers for multivariate models with more than two parameters.

Following ideas from the AutoCrypt 2013 paper “Behind the Scene of Side-Channel Attacks”, we also provide fast computation algorithms in which the traces are accumulated prior to computing the distinguisher values.

[Full version at PROOFS 2016 ▲]

1. Fact
Side-channel leaks are:
- multi-variate
- multi-model

2. Matrix Notations

- \( Q \) : number of queries,
- \( D \) : number of samples,
- \( S \) : number of models.

In matrix notation:

\[
X = \alpha Y + N
\]

where
- \( X \) is a matrix of size \( D \times Q \)
- \( \alpha \) is a matrix of size \( D \times S \)
- \( Y \) (the star means "for the correct key \( k = k^\star \)) is a matrix of size \( S \times Q \)
- \( N \) is a matrix of size \( Q \times D \)

3. Real World Example
The figures below show power consumption traces taken from an ATXmega manufactured datasets from the DPA contest V4 team [7] (knowing the mask):

- (a) Weights of bits of the sensitive variable
- (b) Mean power consumption for each Hamming weight class

4. Question
What is the optimal distinguisher, when in Equation (1): \( \alpha \) is known? \( D_{MIL}(x, t) = \argmin_k || \tilde{x} - y ||^2 \)
- \( \alpha \) is unknown? \( D_{MIL,sto}(x, t) = \argmax_k \sum \hat{Cov}(x'_d, y)^2 / \hat{Var}(y) \)

5. Solution

Theorem 1. The optimal maximum likelihood (ML) distinguisher [?] for Gaussian noise writes

\[
D_{ML}(x, t) = \argmin_k (x - \alpha y)^T \Sigma^{-1} (x - \alpha y)
\]

Proof From [?] we have \( D_{ML}(x, t) = \argmin_k \rho(x|y) \) where from \( \Theta \) it is easily seen that \( \rho(x|y) = p(y|x) \). From the i.i.d. assumption the noise density \( p(y|\Theta) \) is given by

\[
p(y|\Theta) = \frac{1}{\sqrt{(2\pi)^D \det(\Sigma)}} \exp \left( -\frac{1}{2} (y - \mu)^T \Sigma^{-1} (y - \mu) \right)
\]

Thus \( p(y|x) \) is maximum when the expression \( x^T \Sigma^{-1} x \) for \( x = x - \alpha y \) is minimum.

Theorem 2. The optimal stochastic multivariate attack is given by

\[
D_{ML,sto}(x, t) = \argmax_k \left( y^T (yy^T)^{-1} y x^T \Sigma^{-1} x \right)
\]

for which the optimal value of \( \alpha \) is given by

\[
\alpha^{opt} = (xy^T)^{-1} y
\]

Proof Let \( x' = \Sigma^{-1/2} x \) and \( y' = (yy^T)^{-1/2} y \). The optimal distinguisher minimizes the following expression over \( \alpha \in \mathbb{R}^{D \times S} \):

\[
tr \left( (x - \alpha y)' \Sigma^{-1} (x - \alpha y) \right) = tr \left( (x' - \alpha y') (x' - \alpha y')^T \right) - \frac{D}{2} \| x' - \alpha y' \|^2.
\]

The minimization over \( \alpha' \) yields \( \alpha' = (x'y') (yy^T)^{-1} \) for all \( d = 1, \ldots, D \). This gives \( \alpha = (x'y') (yy^T)^{-1} \) hence \( \alpha = (xy^T)^{-1} y \), which remarkably does not depend on \( \Sigma \).

6. Summary for \( S > 2 \) Models

Mathematical expression for multivariate (\( D \geq 1 \)) optimal attacks with a linear combination of models (\( S \geq 1 \))

Leakage model:
- \( x = \alpha y + n \)
- \( y \sim N(0, \Sigma) \)

Optimal distinguisher:

\[
D_{ML}(x, t) = \argmin_k (x - \alpha y)^T \Sigma^{-1} (x - \alpha y)
\]

6bis. Summary for \( S = 2 \) Models

Mathematical expression for multivariate (\( D \geq 1 \)) optimal attacks with one model \( X \) associated to envelope \( \alpha \in \mathbb{R}^{D\times 2} \) and a constant offset \( \beta \in \mathbb{R}^{D\times 1} \) (\( S = 2 \)).

Leakage model:

\[
x = \alpha y + \beta + n
\]

Optimal distinguisher:

\[
D_{ML}(x, t) = \argmin_k (x - \alpha y)^T \Sigma^{-1} (x - \alpha y)
\]

Affine projection:

\[
x' = A x - \beta
\]

Data transformation:

\[
x' = \Sigma^{-1/2} x
\]

Univariate ML attack:

\[
D_{ML}^x(x, t) = \argmin_k |x - y|^2
\]

New multivariate CPA attack:

\[
D_{ML,sto}(x, t) = \argmax_k \sum_{d=1}^D \hat{Cov}(x'_d, y)^2 / \hat{Var}(y)
\]