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To cite this version:
Nicolas Bruneau, Sylvain Guilley, Annelie Heuser, Marion Damien, Olivier Rioul. Optimal attacks for multivariate and multi-model side-channel leakages. 18th Workshop on Cryptographic Hardware and Embedded Systems (CHES 2016), Aug 2016, Santa Barbara, United States. Cryptographic Hardware and Embedded Systems – CHES 2016, 2016. hal-02300058

HAL Id: hal-02300058
https://telecom-paris.hal.science/hal-02300058
Submitted on 20 Aug 2021

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Optimal Attacks for Multivariate and Multi-model Side-Channel Leaksages
Nicolas Bruneau, Sylvain Guilley, Annelie Heuser, Damien Marion and Olivier Rioul

Abstract. In practice, a side-channel signal is measured as a trace consisting of several samples where several sensitive bits are manipulated in parallel, each leaking differently. Therefore, the informed attacker needs to devise side-channel distinguishers that can handle both multivariate leakages and multivariate models at the same time. In the state of the art, these two issues have two independent solutions: on the one hand, dimensionality reduction can cope with multivariate leakage; on the other hand, online stochastic approaches can cope with multivariate models. In this work, we combine both solutions to derive closed-form expressions of the resulting independent solutions: on the one hand, dimensionality reduction can cope with multivariate leakage; on the other hand, online stochastic approaches can cope with multivariate models. Following ideas from the AsiaCrypt’2013 paper "Behind the Scene of Side-Channel Attacks" we also provide fast computation algorithms in which the traces are accumulated prior to computing the distinguisher values. 

1. Fact
Side-channel leakages are:

- multi-variate
- multi-model

(e.g., each bit leaks separately)

2. Matrix Notations

\[ X = \alpha Y + N \] 

where

- \( X \) is a matrix of size \( D \times Q \)
- \( \alpha \) is a matrix of size \( D \times S \)
- \( Y \) (the star means "for the correct key \( k = k^* \)"") is a matrix of size \( S \times Q \)
- \( N \) is a matrix of size \( D \times S \)

3. Real World Example

The figures below show power consumption traces taken from an ATmel manufactured dataset available from the DPA contest V4 team [7] (knowing the mask):

4. Question

What is the optimal distinguisher, when in Equation (4)?

\[ \text{ML} \] (x, t) = argmin \( k \) \( \| x - y \|_2^2 \) 

5. Solution

Theorem 1. The optimal distinguisher \( ML(x, t) \) for Gaussian noise writes

\[ D_{\text{ML}}(x, t) = \text{argmin} \ (x - \alpha y)^T \Sigma^{-1} (x - \alpha y) \] 

Proof From [2] we have \( D_{\text{ML}}(x, t) = \text{argmax}_k p(y|\alpha) \) where from \( p(\alpha) \) it is easily seen that \( p(x|\alpha) = p(x|\alpha) \). From the i.i.d assumption the noise density \( p(x) \) is given by

\[ p(x|\alpha) = \prod_{q=1}^Q \frac{1}{\sqrt{(2\pi)\Sigma}} \exp \left(-\frac{1}{2} x^T \Sigma^{-1} x \right) \]

Following ideas from the AsiaCrypt’2013 paper "Behind the Scene of Side-Channel Attacks", we also provide fast computation algorithms in which the traces are accumulated prior to computing the distinguisher values.

6. Summary for \( S > 2 \) Models

Mathematical expression for multivariate \((D \geq 1)\) optimal attacks with a linear combination of models \((S \geq 1)\):

Leakage model:

\[ x = \alpha y + n \]

\[ y \sim \mathcal{N}(0, \Sigma) \]

\[ y = \phi(t, k) \]

Optimal distinguisher:

\[ D_{\text{ML}}(x, t) = \text{argmin}_k (x - \alpha y)^T \Sigma^{-1} (x - \alpha y) \]

6bis. Summary for \( S = 2 \) Models

Mathematical expression for multivariate \((D \geq 1)\) optimal attacks with a one model \( Y \) associated to envelope \( \alpha \in \mathbb{R}^{D \times 1} \) and a constant offset \( \beta \in \mathbb{R}^{D \times 1} \):

Leakage model:

\[ x = \alpha y + \beta + n \]

\[ y \sim \mathcal{N}(0, \Sigma) \]

\[ y = \phi(t, k) \]

Optimal distinguisher:

\[ D_{\text{ML}}(x, t) = \text{argmin}_k (x - \alpha y)^T \Sigma^{-1} (x - \alpha y) \]