



On optimality of MIA for unknown leakage models and related new practical results

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On the Optimality of Mutual Information Analysis

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Motivation & State-of-the Art

- As the threat of side-channel attacks is well known, countermeasures are used for protection
- For example: Weakening the link between the measured leakage $\mathbf{X}$ and the sensitive variable $\mathbf{y}$
- Generic distinguisher cope with this scenario
 - Mutual Information Analysis (MIA) [1]
 - Kolmogorov-Smirnov distance (KSA) [2]
 - [3],...

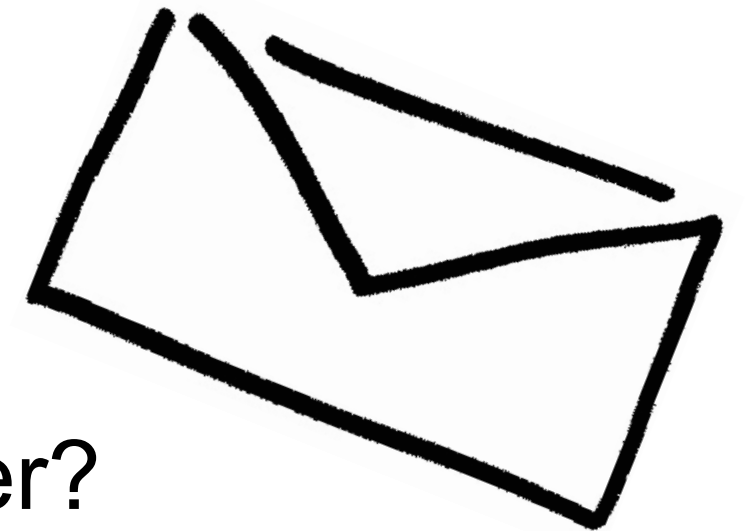
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[2] Nicolas Veyrat-Charvillon and François-Xavier Standaert. Mutual Information Analysis: How, When and Why? In CHES, volume 5747 of LNCS, pages 429–443. Springer, September 6-9 2009. Lausanne, Switzerland.

[3] N. Veyrat-Charvillon and F.-X. Standaert. Generic side-channel distinguishers: Improvements and limitations. In P. Rogaway, editor, CRYPTO, volume 6841 of Lecture Notes in Computer Science, pages 354–372. Springer, 2011.

[4] Julien Doget, Emmanuel Prouff, Matthieu Rivain, François-Xavier Standaert: Univariate side channel attacks and leakage modeling. J. Cryptographic Engineering 1(2): 123-144 (2011)

Take home message!



- What is the *optimal* generic distinguisher?
- By applying the maximum likelihood principle, we derive the optimal generic distinguisher
- When the leakage has been quantified and probabilities are estimated from histograms, the optimal distinguisher's expression turns out to coincide with the mutual information analysis

Universal Maximum Likelihood Equivalent to MIA

Notations & Assumptions

- Values are quantized (discrete leakage)

$$\hat{\mathbb{P}}(x|y) = \frac{\sum_{i=1}^m \mathbb{1}_{x_i=x, y_i=y}}{\sum_{i=1}^m \mathbb{1}_{y_i=y}} = \frac{\hat{\mathbb{P}}(x, y)}{\hat{\mathbb{P}}(y)}$$

$$\hat{\mathbb{P}}(x, y) = \frac{1}{m} \sum_{i=1}^m \mathbb{1}_{x_i=x, y_i=y}$$

$$\hat{\mathbb{P}}(y) = \frac{1}{m} \sum_{i=1}^m \mathbb{1}_{y_i=y}$$

$$\hat{\mathbb{P}}(x) = \frac{1}{m} \sum_{i=1}^m \mathbb{1}_{x_i=x}$$

- Empirical Mutual Information

$$\hat{I}(\mathbf{x}, \mathbf{y}) = \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} \hat{\mathbb{P}}(x, y) \log_2 \frac{\hat{\mathbb{P}}(x, y)}{\hat{\mathbb{P}}(x) \hat{\mathbb{P}}(y)}$$

- From the Maximum Likelihood it is known that maximizing the success rate amounts to select the key guess $\hat{k}$ that maximizes

$$\mathbb{P}(\mathbf{x}|\mathbf{y})$$

- In practice if no profiling is possible the conditional distribution is unknown
- Therefore, we need a *universal* version (computed from the available data without prior information)

$$\hat{k} = \arg \max_k \hat{\mathbb{P}}(\mathbf{x}|\mathbf{y}) = \arg \max_k \prod_{i=1}^m \hat{\mathbb{P}}(x_i|y_i)$$

- Universal Maximum Likelihood is equivalent to mutual information analysis

$$\hat{k} = \arg \max_k \hat{I}(\mathbf{x}, \mathbf{y})$$

MIA is the optimal tool for key recovery when the model is unknown.

Proof sketch

Denoting $n_{x,y} = \sum_{i=1}^m \mathbb{1}_{x_i=x, y_i=y} = m \hat{\mathbb{P}}(x, y)$

$$\hat{\mathbb{P}}(\mathbf{x}|\mathbf{y}) = \prod_{i=1}^m \hat{\mathbb{P}}(x_i|y_i) = \prod_{x \in \mathcal{X}, y \in \mathcal{Y}} \hat{\mathbb{P}}(x|y)^{n_{x,y}}$$

$$\hat{\mathbb{P}}(\mathbf{x}|\mathbf{y}) = \prod_{x \in \mathcal{X}, y \in \mathcal{Y}} \hat{\mathbb{P}}(x|y)^{m \hat{\mathbb{P}}(x,y)} = 2^{-m \hat{H}(\mathbf{x}|\mathbf{y})}$$

$$\text{where } \hat{H}(\mathbf{x}|\mathbf{y}) = \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} \hat{\mathbb{P}}(x, y) \log_2 \frac{1}{\hat{\mathbb{P}}(x|y)}.$$

Empirical Future Work

- Experiments showing empirically the optimality of Mutual Information
- Especially, in comparison to Linear Regression Analysis [4]