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# Success Rate Exponents for Side-Channel Attacks

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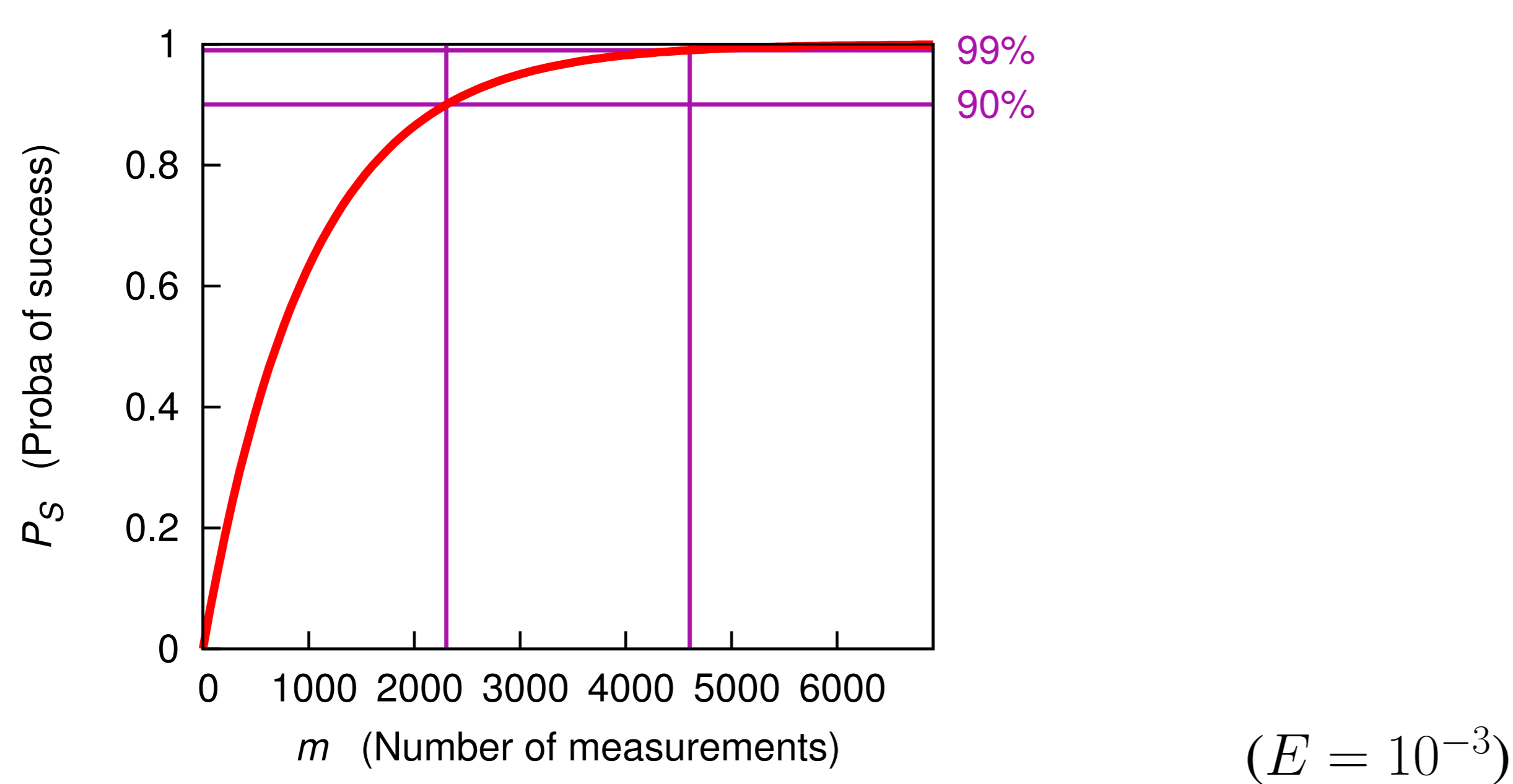
## Goal

- Compute the exact probability of success  $\mathbb{P}_S = \mathbb{P}(\hat{K} = K^*)$
- Rigorous mathematical computation of its first order exponent of success rate:

$$\mathbb{P}_S \approx 1 - e^{-mE} \quad \text{for some } E, \text{ where } m \text{ is the number of measurements.} \quad (1)$$

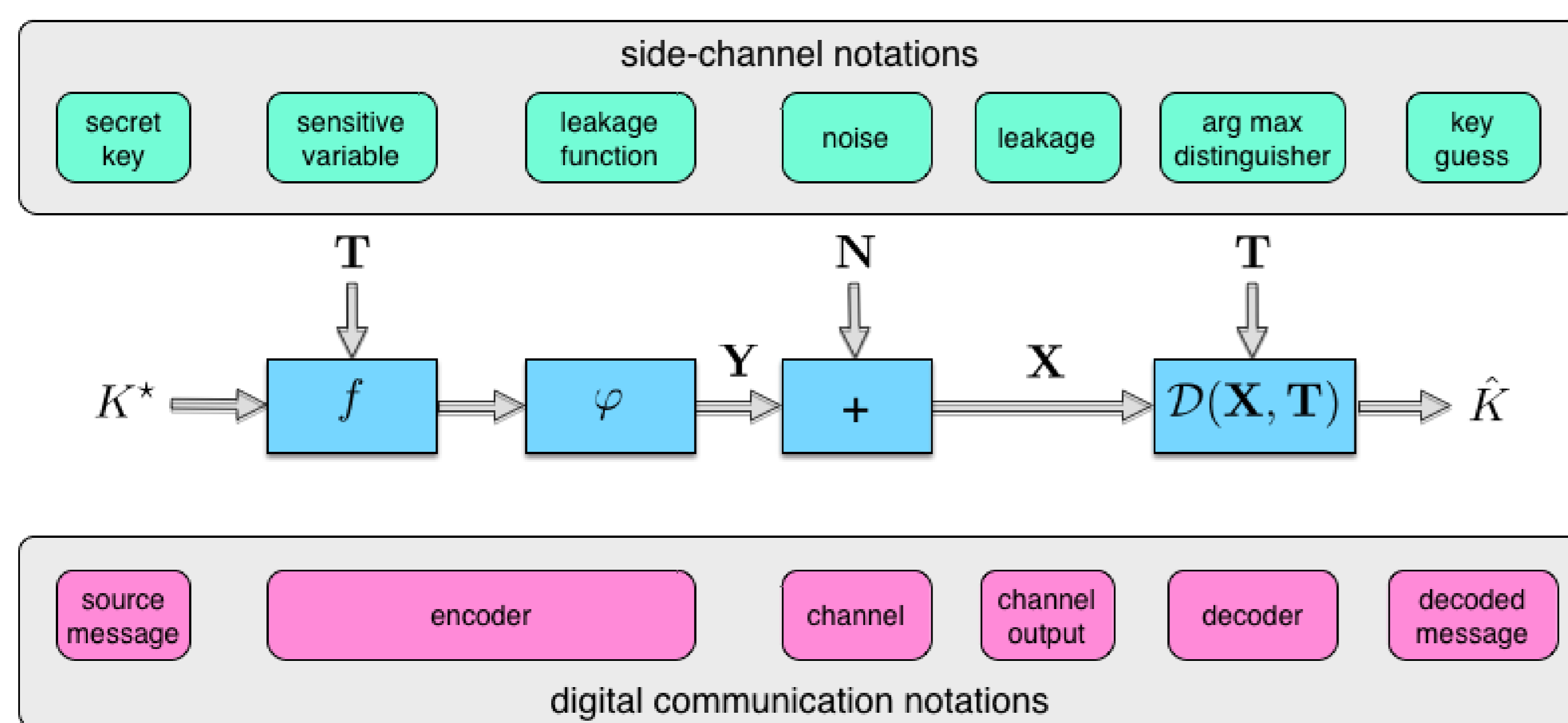
## Example (with $E$ independent of $m$ )

- By Eq. (1), if  $\mathbb{P}_S = 90\%$ , then  $m = \frac{\ln(10)}{E}$ ;
- Doubling the number of measurements  $m \rightarrow 2m \Rightarrow \mathbb{P}_S = 99\%$ .



## Side-channel analysis

as a communication channel [HRG14]



## Relationship with the state-of-the-art

At CRYPTO '99 [CJRR99], Chari, Jutla, Rao and Rohatgi gave a lower bound on the probability of success of mono-bit side-channel attacks. The bound is exponential in the number of queries.

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- [FLD12] Yunsi Fei, Qiasi Luo, and A. Adam Ding. A Statistical Model for DPA with Novel Algorithmic Confusion Analysis. In Emmanuel Prouff and Patrick Schaumont, editors, *CHES*, volume 7428 of *LNCS*, pages 233–250. Springer, 2012.
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## Useful concepts

**Definition 1** (First-Order Exponent Equivalence).

A sequence  $p_m$  of positive numbers admits a first-order exponent  $E_m$  if  $\epsilon_m = E_m + \frac{1}{m} \ln p_m$  tends to zero as  $m \rightarrow +\infty$ . In this case we write:

$$p_m \approx e^{-mE_m}.$$

In practice, “ $m = 20$ ” is already “ $m \rightarrow +\infty$ ”.

## Result for Gaussian noise

When  $Y = \alpha X(k) + N$ , with  $N \sim \mathcal{N}(0, \sigma^2)$  is the noise:

$$E = \frac{1}{8\sigma^2} \min_{k \neq k'} \mathbb{E}(Y(k) - Y(k'))^2 \quad (2)$$

$$= \frac{1}{2} \times \text{SNR} \times \min_{k \neq k'} \kappa_{k,k'}, \quad (3)$$

- where  $\text{SNR} = \frac{\alpha^2}{\sigma^2}$ , and
- where  $\kappa_{k,k'} = \frac{1 - \rho(Y(k), Y(k'))}{2}$  is the confusion coefficient [FLD12].

## Proof (for the optimal distinguisher when the noise is Gaussian, i.e., the opposite norm-2)

$$D_k^{\text{opt}}(\mathbf{x}, \mathbf{t}) = -\|\mathbf{x} - \mathbf{y}(k)\|_2. \quad (4)$$

*Nota bene:* bold letters  $\mathbf{x}$  represent series:  $\mathbf{x} = (x_i)_{1 \leq i \leq m}$ .

We define the *marginal pairwise probability of failure*:

$$\mathbb{P}_{k \rightarrow k' | \mathbf{t}} = \mathbb{P}(\|\mathbf{X} - \mathbf{Y}(k)\| \leq \|\mathbf{X} - \mathbf{Y}(k')\| | \mathbf{T} = \mathbf{t}). \quad (5)$$

**Lemma 1** ( $Q$ -expression of the Pairwise Failure Probability).

Let  $Q(x) = \frac{1}{2} \text{erf}\left(\frac{x}{\sqrt{2}}\right) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2/2} dt$  be the complementary cdf of a standard normal  $\mathcal{N}(0, 1)$ . We have:

$$\mathbb{P}_{k \rightarrow k' | \mathbf{t}} = Q\left(\frac{\|\mathbf{y}(k) - \mathbf{y}(k')\|}{2\sigma}\right).$$

**Lemma 2** (first-order Gaussian  $Q$  exponent).

As  $x \rightarrow +\infty$ ,  $Q(x) \lesssim e^{-x^2/2}$ , in the sense that  $Q(x) \leq e^{-x^2/2}$  for all  $x \geq 0$  and  $\frac{\ln Q(x)}{x^2} \rightarrow -\frac{1}{2}$ .

Dominated convergence theorem:  $\mathbb{E}[\ln \mathbb{P}_{k \rightarrow k' | \mathbf{T}} / m] \rightarrow -\frac{\mathbb{E}(Y(k) - Y(k'))^2}{8\sigma^2}$ , when  $m \rightarrow +\infty$ . Combining yields the result.  $\square$

Our results generalize the closed-form expressions of the success probability recently obtained in the case of normal noise when correlation is used as a distinguisher [FLD12, TPR13, DZFL14]. First order exponents are an alternative to the *relative distinguishing margin* (RDM [WO11]) because:

- they are completely related to the ideal criterion (namely, the success rate) for large (even moderately large) number of measurements  $m$ ,
- they can be computed explicitly for many distinguishers to indicate how performance relates to noise power, model, etc. without resorting to experimentation.