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Abstract—**Irrigation in agriculture is a major source of electricity demand flexibility that goes largely unexploited. In this paper we provide a model and a solution to the problem of scheduling irrigation time to minimize electricity costs while satisfying crop water requirements. We propose to apply rebates (aimed to consume renewable energy surplus) that were traditionally offered to the industrial sector, in the agricultural one. Furthermore an architecture is proposed to overcome some of the limitations that can hinder the adoption of such rebates. The architecture integrates scheduling techniques best studied in the networking literature.**

Numerical analysis is performed to validate our model and evaluate the proposed scheduling mechanisms, based on real data from a soybean producer and from the corresponding electricity operator. Results indicate that significant cost reductions can be obtained, specially if the rebates are considered.

Index Terms—**Flexibility, Smart Grids, Irrigation, Energy Management**

I. Introduction

Nowadays we are facing a global effort to move away from carbon-based energy to renewable energy resources. This is evidenced in programs such as Europe's 2050 [1]. The fact that power grids were not designed to support the large amount of injection in the low voltage system is hindering the massive adoption of distributed energy resources (DER).

One of the most popular strategies for integrating DER is to dynamically exploit demand flexibility through Demand Response (DR) programs. These were initially reserved for big industrial players [2], but thanks to the application of Information and communication technologies under the smart grid paradigm, they have been recently extended to smaller commercial or residential consumers. One prominent example is the vast literature in Local Energy Markets [3], [4], [5], which seek to incentivize the consumption of locally produced energy [6].

In this paper we deal with a third, mostly unstudied, case: demand response for medium consumers such as farmers with specific necessities, e.g. satisfying crop water requirements.

Indeed, participating in flexibility programs is particularly interesting for farmers, for whom the cost of electricity used in irrigation systems can account for up to 30% of the total production costs. Moreover, irrigation is mostly done without automatic control, or without considering both real time irrigation requirements and energy prices. This is an important problem, and several novel solution approaches have been considered [7][1]. Furthermore, it has also been shown that irrigation increases crop yield, even in countries with temperate climates [8].

Irrigation can be an important source of flexibility [9] and crop productivity depends significantly on the structure of irrigation cycles [10]. We point out that the flexibility provided by irrigation is of a special kind: the peak of inflexible load does not occur every single day, but is concentrated on some particular weeks of the crop growth cycle. For this reason, tariffs such as Time-of-Use with intra-day price are not well adapted.

In some countries with high penetration of renewables, the concept of rebates has been introduced. It is a discount in the electricity price to incentivize consumption when there is a surplus of generation.

The goal of this study is to investigate the extent of the benefits that can be obtained by exploiting flexibility in these cases. In doing so, we bridge the real practices in agriculture with innovative energy market models in an effort to fully utilize latent flexibility.

The idea of decreasing the usage of electricity in irrigation is not new [11], [12]. In [11], the authors optimize the water pressure of the irrigation system, while [12] considers the flexible operation of an irrigation system. None of these studies takes into account either the benefits of exploiting flexibility by the electricity operator nor the reduction in the electricity cost by properly utilizing electricity tariffs.

The contributions of this paper are summarized as follows:

- We model the energy management problem with irrigation requirements as constraints and provide and algorithm to solve it. Evaluation is conducted on real data, achieving significant reductions in the electricity cost incurred by farmers while keeping the optimal level of productivity.
- We evaluate the effect of dynamic rebates based on renewable energy surplus in irrigation scheduling. It is shown that these can be beneficial to both the farmers and the utility.
- Scheduling techniques are proposed to solve the problem of assigning a limited amount of surplus to an increasing number of participants in a fair manner.

[1] http://www.irricontrol.com.uy/Irricontrol

In this section we introduce the nomenclature used for formulating the model in the next section.

O_t	Root zone depletion (RZD) (mm)
G_t	Effective rain (mm)
H_t	Effective evapotranspiration (mm)
F_t	Nominal evapotranspiration (mm)
Q_t	Water content in the ground (WCG) (mm)
η	Efficiency of the irrigation system
ρ	Water supplied per irrigation time (mm/h)
S_t	Irrigation time (h)
J_t	Permanent Wilting Point (PWP) (mm)
L_t	Tiredness fraction
K_t	Total available water (TAW) (mm)
I_t	Field capacity (FG) (mm)
N_t	Level of hydric stress (mm)
M_t	Readily available water (RAW) (mm)
$[x]^+$	Positive part of x
D	Number of days considered
$[N]$	The natural numbers smaller than N: $\{0, 1, \ldots, N-1\}$

II. PROBLEM DEFINITION

A. Crop Model

A task of vital importance in Agriculture is the irrigation of fields, due to its positive impact on yields. The water supplied to the crops should be enough to maximize the production but no more than that. This turns out to be a difficult task as the water requirement of a crop varies depending on the stage of growth. Climate conditions also play an important role in determining these requirements.

A plant absorbs most of its water requirements from its roots at a precise depth that varies with plant growth. The amount of water available in the ground varies between the maximum capacity of the field to retain water (FC) and the permanent wilting point (PWP) at which the plant dies. The total available water within these margins is (TAW), but the plant can only absorb a fraction that is known as readily available water (RAW, Equation (2)). These values are depicted in Figure 1.

The Root Zone Depletion (RZD, Equation (8)) is the level of water that has been depleted from the field capacity and is no longer available for the plant. It depends on its previous value, minus all the irrigation (U, Equation (4)), minus all the rain (G) plus the evaporation in the soil and the transpiration of the plant. These last two phenomena are represented together as the evapotranspiration (H, Equation (6)). The evapotranspiration is piece-wise linear and has two levels depending on the stress of the ground (percentage of RZD with respect to the PWP) [13]. To keep the crop from under-performing (i.e., producing less kilograms per hectare), the Root Zone Depletion needs to be smaller than the readily available water level. Otherwise, the water content in the ground would not be at a level available for the plant to easily reach, causing hydric stress and consequently yield reduction. In Figure 1, two different values of RZD and the corresponding WCG are
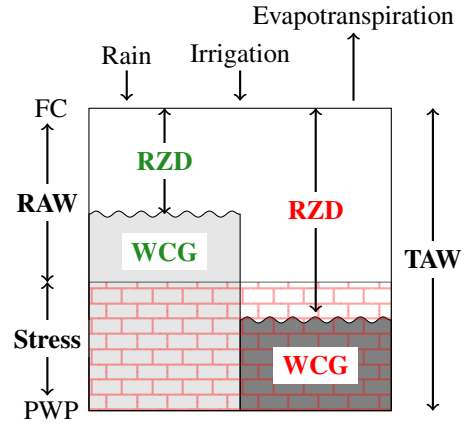

Fig. 1. Illustration of the different parameters in the model.

represented. WCG values are shown as the fully grey shaded areas increasing upwards and the rectangle with the brick pattern represent the level below which water is not easily available for the plant. The red value of RZD is larger than the RAW value, therefore some productivity will be loss. On the contrary, the green RZD value is smaller than the RAW level and there is no loss in production. An equivalent way of stating the problem, which will be used later, is the following: the water content in the ground (WCG, Equation (9)) which is the amount that is left after removing from the FC the water that has been depleted (RZD), has to be larger than the "refilling point", i.e, the amount of water that is left after removing from the FC the RAW. We denote this latter quantity as the level of hydric stress (N, Equation (3)) or as the refilling point.

$$K_t = I_t - J_t \tag{1}$$

$$M_t = L_t K_t \tag{2}$$

$$N_t = I_t - M_t \tag{3}$$

$$U_t = \eta \times \rho \times S_t \tag{4}$$

$$H_0 = F_0 \tag{5}$$

$$H_t = F_t \min\{1, \frac{[Q_{t-1} + G_t + U_t - J_t]}{K_t(1 - L_t)}\} \tag{6}$$

$$O_{-1} = 0 \tag{7}$$

$$O_t = [O_{t-1} - G_t + H_t - U_t]^+ \tag{8}$$

$$Q_t = I_t - O_t \tag{9}$$

In Equation (4), S_t denotes the amount of hours used for irrigation in day t. For convenience, most of the time we will use, instead Y_{tj}, the percentage of the time that the irrigation system was working during the j hour of the day t. It naturally holds that $\sum_j Y_{tj} = S_t$.

B. Electricity tariffs

First, we consider a three-period Time of Use (ToU) tariff as described in Equation (10). There, p_l, p_m, p_h denote the low, medium and high prices (correspondingly), P is the

power (kW) consumed by the pump used for irrigation and $T_l, T_m, T_h \subset [24]$ correspond to the hours in which each of the rates is available.

$$C_1(Y_{ij}) = \begin{cases} Pp_l, & \text{if } j \in T_l, \forall i \\ Pp_m, & \text{if } j \in T_m, \forall i \\ Pp_h, & \text{if } j \in T_h, \forall i \end{cases} \quad (10)$$

Second, we consider a family of rebates on top of the three period ToU described by Equation (10), designed to incentivize consumption whenever there is surplus of generation. Equation (11) defines these rebates, which we will denote Opportunity offers (OO) in the rest of the paper.

$$C_2(Y_{ij}) = \begin{cases} Pp_l, & \text{if } Y_{ij} \leq \alpha_{ij}, \ j \in T_l, \forall i \\ P\beta_l p_l, & \text{if } Y_{ij} > \alpha_{ij}, \ j \in T_l, \forall i \\ Pp_m, & \text{if } Y_{ij} \leq \alpha_{ij}, \ j \in T_m, \forall i \\ P\beta_m p_m, & \text{if } Y_{ij} > \alpha_{ij}, \ j \in T_m, \forall i \\ Pp_h, & \text{if } Y_{ij} \leq \alpha_{ij}, \ j \in T_h, \forall i \\ P\beta_h p_h, & \text{if } Y_{ij} > \alpha_{ij}, \ j \in T_h, \forall i \end{cases} \quad (11)$$

Opportunity offers work as follows: for a given hour and day (i, j), if the consumption Y_{ij} is greater than a threshold α_{ij} that emulates average past consumption, the cost to be paid is a fraction $\beta_\square \in (0, 1]$ of the original cost. If no OO is available, α_{ij} can be set to 1, which yields the same result. The variables $P, p_l, p_m, p_h, T_l, T_m, T_h$ have the same interpretation as in Equation (10).

C. Optimization Problem

The optimization problem defined in Equation (12) seeks to find an irrigation assignment guaranteeing the required level of water in the ground and minimizing the cost of irrigation. The two different tariffs (Equations (10),(11), represented by $C_\square$) will be used as cost functions.

$$\min_Y \ \sum_{t,j} C_\square(Y_{tj}) \quad (12a)$$

$$\text{s.t.} \quad O_t \leq M_t, \quad t \in [N], \quad (12b)$$

$$\sum_j Y_{tj} = S_t, \quad t \in [N], \quad (12c)$$

$$Y_{tj} \in [0,1], \ t \in [N], \ j \in [24] \quad (12d)$$

Observe that constraint 12b is equivalent to $\{Q_t \geq N_t\}$, The function $O_t \colon [0,1]^{24 \times D} \to \mathbb{R}$ is not differentiable and, for some combinations of the parameters, the set $\{O_t \leq M_t\}$ is not convex. Figure 2 depicts one such case. This motivated the development of an algorithm to solve the irrigation scheduling problem, as presented in the next section.

III. Proposed Solution

A. Algorithm

In this section, we present an algorithm to solve the optimization problem (12). A key idea of our solution is to
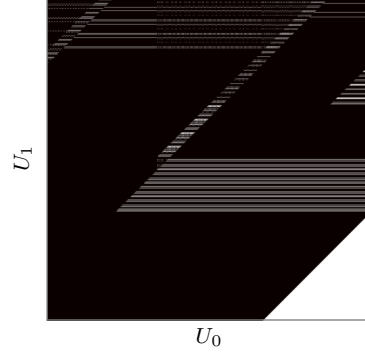

Fig. 2. The set of points satisfying $O_1 \leq M_1$ for some combination of parameters. Pixels in white represent points in the set and in black, outside. This set is clearly non-convex.

notice that, to satisfy each of the constraints "$O_t \leq M_t$", only variables Y_{ij} with $i \leq t$ could be used. That is, only the variables Y_{1j} affect O_1, while for the constraint $O_2 \leq M_2$, only Y_{1j}, Y_{2j} can be used, etc. Therefore, we can make the problem feasible by using only those variables.

With that observation in mind, the algorithm is fairly simple, namely: consume the least amount of water to satisfy all the constraints and try to use the variables with the lowest cost associated with them.

Algorithm 1 Optimization Algorithm

Input: F, G, H, J, K, L, $C_\square$, ϵ
Output: Y
    $Y_{ij} \leftarrow 0, \ \forall i, j$
1: **for** $i = 1$ to T **do**
2:     **while** $O_i > M_i$ **do**
3:         $l, k \leftarrow$ *select best variable* % decrease O with least cost

4:         $Y_{lk} \leftarrow Y_{lk} + \epsilon$
5:         *update* O_i
6:     **end while**
7: **end for**
8: **return** Y

To decide which variable should be used, Algorithm 2 was employed.

B. Opportunity assignation

One of the aims Opportunity offers were created for is to sell the surplus of renewable generation.

As they are implemented now, a massive adoption of these tariffs could result in a peak of consumption greater than the original surplus. If such a case arises, more expensive units will have to be dispatched to satisfy the new demand, resulting in additional costs and a negative environmental impact.

In this subsection we propose four mechanisms inspired from the rich literature in scheduling to allocate the surplus of renewable energy without exceeding the available quantities.

Algorithm 2 Selecting the best variable

Input: F, G, H, J, K, L, t,
Output: z, w

    $z, w, \partial_{zw} \leftarrow -1, -1, 0$
1: **for** $i = 1$ to t **do**
2:    **for** $j = 1$ to 24 **do**
3:       $d \leftarrow \frac{\partial O_t}{\partial Y_{ij}}$ (Numerically estimated)
4:       **if** $Y_{ij} \leq 1 - \epsilon$ *and* $d > 0$ **then**
5:          **if** $\alpha_{ij} < 1$ and $\alpha_{zw} = 1$ **then**
6:             $z, w, \partial_{zw} \leftarrow i, j, d$
7:          **else if** $\alpha_{ij} = 1$ and $\alpha_{zw} < 1$ **then**
8:             do nothing
9:          **else**
10:             **if** $\dfrac{\partial_{zw}}{C_2(Y_{zw})} > \dfrac{d}{C_2(Y_{ij})}$ **then**
11:                $z, w, \partial_{zw} \leftarrow i, j, d$
12:             **end if**
13:          **end if**
14:       **end if**
15:    **end for**
16: **end for**
17: **return** (z, w)

All the mechanisms follow the same time structure. For a given opportunity offer occurring at time t of day d, interest of buying from the consumers will be collected the day before $(t, d-1)$ by the scheduler manager (SM). Intention to participate in an offer is expressed by the different participants as a pair $(q^i(t, d), c^i(t, d))$ where $q(t, d)$ represents the consumption in kWh that the participant is requiring to buy and $c^i(t, d)$ is the cost (estimated by each participant) of not irrigating at all during the time t of day d.

Given the offers, the SM decides how much of the requested quantity each participant is allowed and communicates this data back to them. They are then free to reschedule their consumption as they see fit with the new change in tariff. It is important to notice that losing participants are not forbidden to irrigate during hour t of day d. Although this might sound counter-intuitive, we are assuming that the proposed mechanism replaces the assignment of the opportunity offers but leaves the basic tariff structure unchanged and therefore, they are always allowed to use it.

The four evaluated mechanisms, namely: *Least Served First* (LSF), *Most valuable first* (MVF), *Proportional* (PR) and *Fixed priority* (FP), differ in how the SM assigns the opportunity offers, as follows. In LSF, the Scheduling Manager maintains an historic record of the energy assigned to each player in the past. Using this information, she sorts the received offers and assigns all the requested energy to the first participant. If there is any remaining surplus, it continues the allocation following the created order. Tiebreaks are handled arbitrarily. The second approach, *Most valuable first*, assigns as much energy as possible to the player that reported the highest cost $c^i(t, d)$, and if there is any remaining energy, it continues with the second highest cost, etc. The *Proportional*

mechanism consists on distributing the available energy among all participants, proportionally to their submitted offers. Finally, FP is a very simple mechanism included for comparison purposes, in which there is a **fixed** order and energy is **always** distributed according to the same order. Algorithm 3 describes the common feature of the LSF, MVF and FP mechanisms. Their difference lies in the **SortUsingMechanism** function and how the permutation π is built. In Algorithm 3, a^i is the assigned quantity to player i. For the *Proportional* case, the assigned quantity is $a^i = \frac{A(t,d)q^i(t,d)}{C}$, where $A(t, d)$ is the available quantity of surplus, $q^i(t, d)$ is the quantity asked by the farmer i, and $C = \sum_j q^j(t, d)$.

Algorithm 3 Scheduling mechanism

Input: $(q^0(t_k, d_j), c^0(t_k, d_j)), \ldots (q^{N-1}(t_k, d_j), c^{N-1}(t_k, d_j))$
Input: A
Output: $a^0(t_k, d_j), \ldots, a^{N-1}(t_k, d_j)$
1: $\pi \leftarrow$ SortUsingMechanism$([N])$
2: $m \leftarrow 0$
3: $a^l \leftarrow 0, \ l \in [N]$
4: **while** $A > 0$ and $i < N$ **do**
5:    $i \leftarrow \pi(m)$
6:    **if** $q^i(t_k, d_j) > A$ **then**
7:       $a^i \leftarrow q^i$
8:    **else**
9:       $a^i \leftarrow A$
10:    **end if**
11:    $A \leftarrow A - q^i(t_k, d_j)$
12:    $m \leftarrow m + 1$
13: **end while**
14: **return** $a^0, a^2, \ldots, a^{N-1}$

IV. NUMERICAL ANALYSIS

In this section, using real data, we shall demonstrate the benefits of implementing the proposed solutions.

We had access to real irrigation data from a soybean producer in Uruguay. The dataset contains the irrigation profile during 140 days, namely from 9th November 2017 to 28th March 2018, as well as all the other parameters required by the model[2]. The data corresponds to one irrigation pivot used in a field of 75 hectares. The pump required to power the pivot consumes 77kW. We assume that there are no associated costs for starting or stopping the pump. Table I summarizes the parameters used to instantiate the two cost functions except for the thresholds α_{ij}, which where estimated out of historical data as the average consumption in the same day.

First, we evaluate the net benefits of using Algorithm 1, when the tariff structure remains unchanged, i.e., only considering the three-tier ToU. Next, the benefits of including the opportunity offers are measured. Finally, the different mechanisms to assign such OOs are evaluated.

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[2]Some of these had to be estimated using the temperature and rain data during those days.

A. Optimization algorithm using real data

Figure 3 depicts the evolution of the water content in the ground (WCG) for that period, given the irrigation pattern followed by the farmer. It can be seen that the WCG drops below the level of hydric stress, implying that some performance (in terms of kilograms per hectare) was lost. Moreover, the farmer did not manage to fully avoid irrigation during peak periods. Figure 4 shows the distribution of irrigation hours along ToU price periods, where green, yellow and red stand for low, medium and high prices, respectively.
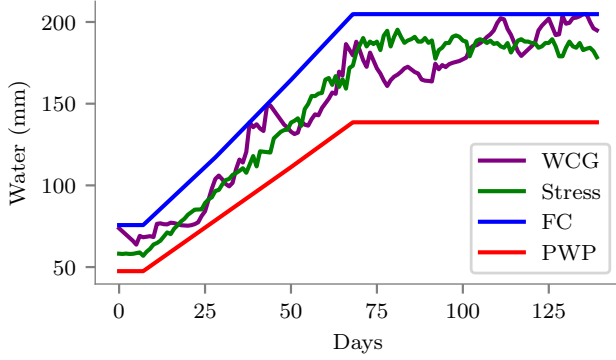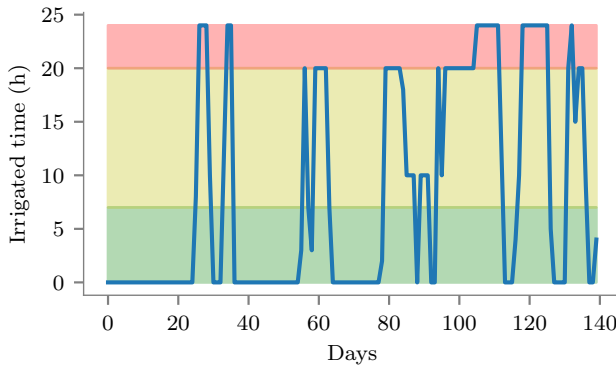

Fig. 3. Irrigation model on real data

Fig. 4. Consumption across time of uses.

For the same dataset, Figure 5 represents the WCG evolution when using the irrigation schedule proposed by our algorithm. Under this new irrigation strategy, the WCG is always above the level of hydric stress and crop performance is optimized. There are models that would allow us to quantify the gains obtained by increasing the performance, but these are outside the scope of this work. Finally, Figure 6 presents the irrigation hours in comparison with the ToU prices for our proposal. Observe that our irrigation strategy respects the constraints imposed by plant water requirements while avoiding the most expensive ToU period.
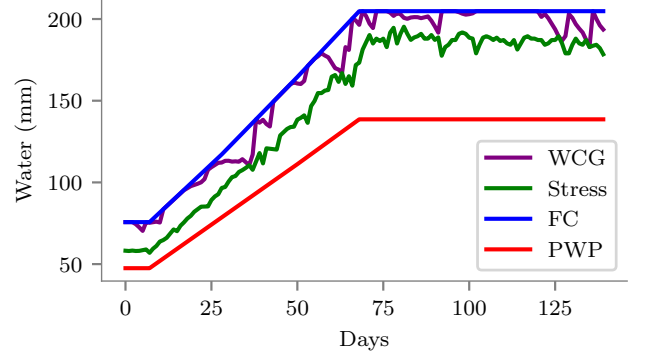

Fig. 5. Algorithm maximizes TOU without taking into account opportunities.
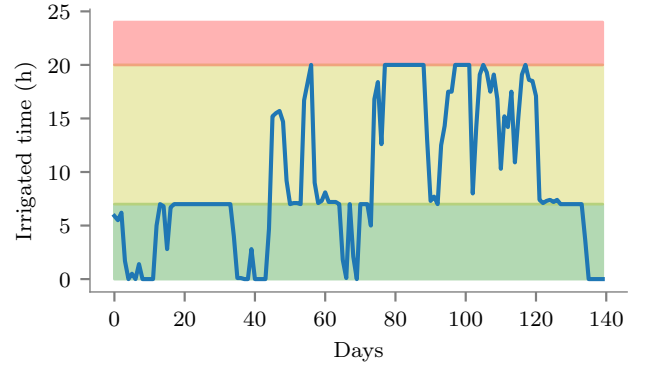

Fig. 6. Algorithm maximizes TOU without taking into account opportunities. Consumption across time of uses.

Table II summarizes the numerical comparison between the cases. Although our algorithm irrigates for a longer period of time (to satisfy the problem constraints), it has an overall decrease in the variable cost[3].

B. Optimizing for Opportunity offers

From the real data on electricity consumption and production surplus event, we obtained the α_{tj} coefficients for the

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[3]The cost of water for most farmers in Uruguay is negligible, particularly when compared with electricity costs and performance losses. Therefore, an increase in the irrigation time, and consequently in the water usage, does not impact the overall irrigation costs.

cost function C_2 defined in Equation (11). As expected, during most of the hours there were no surplus events, and thus no OOs. Figure 7 depicts the hours at which the value of α_{tj} was less than 1, i.e., there was an active rebate. Sub-figure 7.A depicts the consumption of energy with respect to the active offers for the real past consumption. It should be observed that, although the farmer did not plan her irrigation around these rebates (as they are not currently available to farmers), there is some natural overlap between the two. The middle and bottom sub-figures (B and C) also depict the irrigation for those hours with active OOs, but for our algorithm, using C_1 and C_2 as the cost functions, respectively. Naturally, optimizing using C_2 as the cost function increases the irrigation whenever there is a surplus[4].
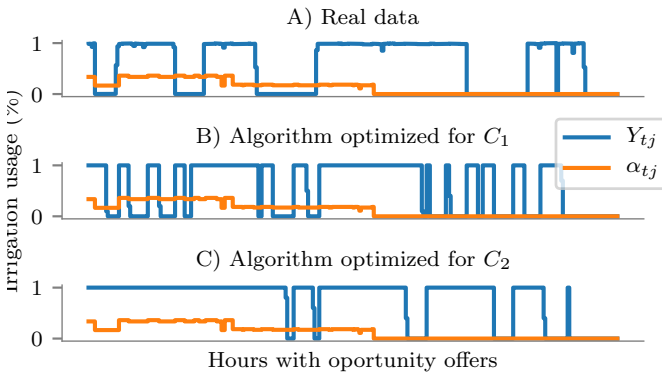

Fig. 7. Consumption during the Opportunity offers for the three considered cases.

The different costs of irrigation if Opportunity Offers were available are presented in Table III. We observe that optimizing for C_1 or C_2 uses the same irrigation time (and therefore the same amount of water), but reductions are bigger: a 15% reduction could be achieved in the costs if Opportunity offers were to be allowed to farmers. This is beneficial for farmers, because they decrease their costs, and also for the utility, because they have extra means to consume the surplus of renewable generation. This can foster the installation of new renewable energy sources in the grid.

C. Evaluation of opportunity assignments

In the previous subsection we showed that offering dynamic rebates to farmers can increase their profit and reduce the amount of renewable energy that need to be curtailed. As mentioned, offering this tariff to new actors can have an associated drawback: if the new peak in demand is greater than

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[4]To solve for C_1, we set $\alpha_{tj} = 1, \ \forall t, j$.

the original surplus, the utility company might suffer higher costs than from just curtailing renewable energy sources. To overcome this difficulty, we evaluate different scheduling mechanisms that aim to assign precisely all the surplus.

To asses the behaviour of the different mechanisms in combination with our algorithm, a synthetic dataset was created from real data. Under the assumption that farmers that are physically close experience similar weather conditions, we took the data we already had and added a small random noise to emulate spatial difference. For the OOs dataset, we used the same periods (t, d) as in the previous sections, yet now we generated the amounts of surplus. In each of these dates, a random quantity of excess of energy was considered in a way that reveals the effects of the different mechanisms: too much energy would satisfy all customers and the mechanisms would be indistinguishable and the same would happen if there is no surplus at all.

Figure 8 illustrates the results of running the different scheduling mechanisms with 20 participants. Each bar plot represents the relative gain of each player (cost with OOs divided by cost without OOs). The number in the legend shows the net cost of all the players averaged over all simulation runs. We seek a mechanism that minimizes the total cost while keeping the variance of different players small, i.e, maximizes fairness.

As expected, the *Fixed Priority* technique was the most unfair (highest variance), followed by the MVF. The least total welfare was obtained by *Least Served First*, closely followed by MVF. Overall, *Least Served First* yielded the best fairness and overall welfare.
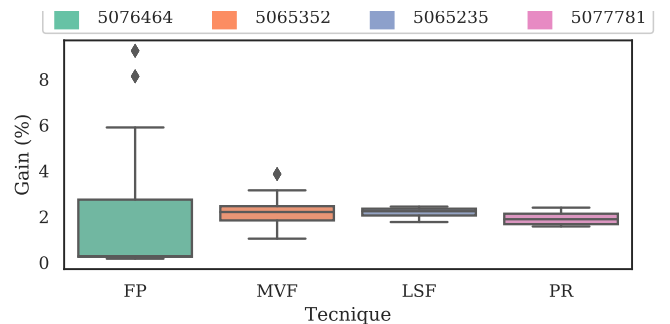

Fig. 8. Gains of farmers under different assignment techniques.

V. DISCUSSION AND FINAL REMARKS

We proposed an architecture and specific irrigation scheduling and coordination mechanisms that enable to exploit electricity demand flexibility from irrigation and, at the same time,

minimize the farmers' electricity costs. Indeed, the proposed scheduler manages to reduce the cost of irrigation electricity while keeping optimal levels of productivity, by exploiting existing ToU tariffs.

Based on real data, we showed that producers do not irrigate in an optimal way and they could greatly benefit from an automatic scheduler, such as the one proposed in this study, in order to boost their production and decrease their energy costs. Moreover, there is a clear potential to exploit opportunity offers, as shown by the decrease of cost in both instances of the algorithm. We demonstrated that the biggest barrier preventing the massive adoption of opportunity offers can be circumvented with our proposed allocation mechanisms.

Consequently, our proposed irrigation scheduler and flexibility allocation mechanisms increase the potential of the grid to host renewable energy sources while reducing the electricity costs in agriculture, which is of key economic relevance in several countries.

We conclude by mentioning some potential improvements that could be achieved through future work. First, the current algorithm is deterministic and requires a forecast of the temperature and precipitation to work with. A natural extension is to pair it with a forecast and use Model Predictive Control to correct the irrigation plan as the real information becomes available. Second, although a Time of Use together with the rebates can provide important incentives, further work should be carried to understand whether a local energy market can provide better results.

References

[1] "Europe's energy roadmap 2050," https://ec.europa.eu/energy/sites/ener/files/documents/2012_energy_roadmap_2050_en_0.pdf, accessed: 2019-04-26.

[2] P. Cappers, C. Goldman, and D. Kathan, "Demand response in u.s. electricity markets: Empirical evidence," *Energy*, vol. 35, no. 4, pp. 1526 – 1535, 2010, demand Response Resources: the US and International Experience. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0360544209002461

[3] E. Mengelkamp, J. Grttner, K. Rock, S. Kessler, L. Orsini, and C. Weinhardt, "Designing microgrid energy markets: A case study: The brooklyn microgrid," *Applied Energy*, vol. 210, pp. 870 – 880, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S030626191730805X

[4] J. Guerrero, A. C. Chapman, and G. Verbi, "Decentralized p2p energy trading under network constraints in a low-voltage network," *IEEE Transactions on Smart Grid*, pp. 1–1, 2018.

[5] J. Horta, D. Kofman, D. Menga, and A. Silva, "Novel market approach for locally balancing renewable energy production and flexible demand," in *2017 IEEE International Conference on Smart Grid Communications (SmartGridComm)*, Oct 2017, pp. 533–539.

[6] D. Kiedanski, D. Kofman, J. Horta, and D. Menga, "Strategy-proof local energy market with sequential stochastic decision process for battery control," in *IEEE Innovative Smart Grid Technologies 2019 NA*, Washington DC, United States, Feb. 2019. [Online]. Available: https://hal.telecom-paristech.fr/hal-02083472

[7] Y. Osroosh, R. Peters, C. Campbell, and Q. Zhang, "Comparison of irrigation automation algorithms for drip-irrigated apple trees," *Computers and Electronics in Agriculture*, vol. 128, pp. 87–99, 10 2016.

[8] F. Montoya and A. Otero, "Is irrigating soybean profitable in uruguay? a modeling approach," *Agronomy Journal*, vol. 111, 01 2019.

[9] P. Barooah, A. Buic, and S. Meyn, "Spectral decomposition of demand-side flexibility for reliable ancillary services in a smart grid," in *2015 48th Hawaii International Conference on System Sciences*, Jan 2015, pp. 2700–2709.

[10] A. Goksoy, A. Demir, Z. Turan, and N. Dagustu, "Responses of sunflower (helianthus annuus l.) to full and limited irrigation at different growth stages," *Field Crops Research*, vol. 87, no. 2-3, pp. 167–178, 2004.

[11] J. R. Daz, R. L. Luque, M. C. Cobo, P. Montesinos, and E. C. Poyato, "Exploring energy saving scenarios for on-demand pressurised irrigation networks," *Biosystems Engineering*, vol. 104, no. 4, pp. 552 – 561, 2009. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1537511009002669

[12] J. L. Merriam, S. W. Styles, and B. J. Freeman, "Flexible irrigation systems: Concept, design, and application," *Journal of irrigation and drainage engineering*, vol. 133, no. 1, pp. 2–11, 2007.

[13] R. Allen, R. Allen, Food, and A. O. of the United Nations, *Crop evapotranspiration: guidelines for computing crop water requirements*, ser. FAO irrigation and drainage paper. Food and Agriculture Organization of the United Nations, 1998, no. nos. 56-57. [Online]. Available: https://books.google.fr/books?id=42hRAAAAMAAJ